

Adaptive Signal Decomposition of Hardy Space

by

Shuang Li

Doctor of Philosophy in Mathematics

2013



Faculty of Science and Technology

University of Macau



Adaptive Signal Decomposition of Hardy Space

by

Shuang Li

Supervisor: Tao Qian

Department of Mathematics

Doctor of Philosophy in Mathematics

2013

Faculty of Science and Technology

University of Macau



Author's right 2013 by
LI, Shuang





Acknowledgments

I wish to express my hearty and deepest gratitude to my supervisor, Professor Tao Qian, for his inspiring guidance, constant encouragement and help throughout the whole period of my Ph.D studies, and also for his genuine concern to my academic development. He proposed to me the significant thesis subject and provided me unflagging support both morally and financially in pursuing it. His encouragement, support and advice have been immensely valuable, both in personal and professional terms.

I would also like to acknowledge other scholars for their valuable suggestions, their warm-hearted assistance and support to my Ph.D studies. Discussions with them contributed to my understanding to some aspects of signal analysis, as well as, to the application aspect of established theory. They include Prof. Luoqing Li, Prof. Qiuhui Chen, Prof. Hong Li, Prof. Yang Chen, Prof. Pengtao Li, Prof. Sik-Chung Tam, Prof. Kit-Ian Kou, Prof. Uwe Kähler, Prof. Yik-Man Chiang, Prof. Ieng-Tak Leong, Prof. Xingdi Chen, Dr. Pei Dang, Dr. Yanbo Wang and Dr. Wen Mi. I have also benefited from time spent with friends at University of Macau. I would also like to thank Dr. Jinxun Wang, Dr. Zhulin Liu, Dr. Yan Mo, Dr. Haichou Li and Dr. Weixiong Mai.

Finally, I owe a great debt to my parents for their constant encouragement, support and endless love.

Abstract

In this thesis, we study adaptive signal decomposition in Hardy space $H^2(\mathbb{D})$. It is a continuation of AFD (Adaptive Fourier Decomposition) theory, which was proposed by Prof. Tao Qian recently. The core idea of AFD is to decompose an analytic signal into basic blocks called mono-components, which have positive instantaneous frequencies. AFD originated from significant observation of Blaschke product and the “Maximal Selection Principle” by Prof. Qian et al.

The thesis is arranged as follows. Chapter 1 is dedicated to the background of the general theory. In Chapter 2, we briefly recall Hardy space $H^2(\mathbb{D})$ and basic signal processing techniques such as Matching Pursuit, Basis Pursuit and AFD. Based on Takenaka-Malmquist (TM) system, AFD finds approximations by consecutively selecting the poles for the TM basis functions in the sense of energy. We also introduce our original work of nonharmonic system $\{e^{i\lambda t}\}_{\lambda \in \mathbb{C}}$ and its application to solving differential equations, which is also Matching-Pursuit-derived method. In Chapter 3, we summarize the recent theory of Compressed Sensing technique. Introduced in 2005 by David Donoho and the group of Terence Tao, Emmanuel Candes and Justin Romberg, the Compressed Sensing theory has attracted much attention in mathematics and in the engineering fields. It gives sparse recovery of signals and images in the sense of probability. However, the Null Space Property (NSP) and the Restricted Isometry Property (RIP) are difficult to test. The situation has not been improved until the random matrix theory was taken into consideration. The latest study on sparse representations via dictionaries by Compressed Sensing is discussed. In Chapter 4, we mainly analyze the asymptotic singular values distribution of reproducing kernel dictionary. These results come from our original papers in this field. They explain why the reproducing kernel dictionary is suitable for sparse reconstruction of Hardy signals. Chapter 5 is dedicated to the application of our theory. We give two examples to illustrate our algorithm. In addition, we apply our method to system identification which is also an aim of AFD.

Declaration

I declare that the thesis here submitted is original except for the source materials explicitly acknowledged and that this thesis as a whole, or any part of this thesis has not been previously submitted for the same degree or for a different degree.

I also acknowledge that I have read and understood the Rules on Handling Student Academic Dishonesty and the Regulations of the Student Discipline of the University of Macau.



Contents

List of Figures	v
List of Tables	vi
1 Introduction	1
1.1 General Background	1
1.2 Specific Background	3
1.3 Research Goals and Objectives	3
1.4 Research Methodology and Design	4
1.5 Potential Contributions	5
1.6 Organization of the Thesis	6
1.7 Statement of Originality	6
2 Hardy Space and Algorithms	8
2.1 Hardy Space	8
2.2 Fourier analysis	12
2.3 Algorithms	13
2.3.1 Matching Pursuit	14
2.3.2 Nonharmonic System	16
2.3.3 AFD of $H^2(\mathbb{D})$	22
2.3.4 AFD of $H^2(\Pi)$	25
2.3.5 Basis Pursuit	28
3 Compressed Sensing Theory	32
3.1 Background	32
3.2 Preliminaries and Notations	33
3.3 Null Space Property and Restricted Isometry Property	35
3.4 Random Matrix	38
3.5 Near Optimal Recovery	40

3.6	Compressed Sensing with Dictionary	42
4	Reproducing Kernel Dictionary	44
4.1	Introduction	44
4.2	Reproducing Kernel Dictionary	44
4.2.1	Case 1	49
4.2.2	Case 2	52
4.2.3	Case 3	57
5	Numerical Examples and Applications	61
5.1	Introduction	61
5.1.1	Example 1	63
5.1.2	Example 2	64
5.1.3	System Identification	65
5.1.4	Example 1	67
5.1.5	Example 2	68
6	Conclusions and Future Work	70
6.1	Conclusions	70
6.2	Limitations of Current Study	70
6.3	Perspectives for Future Work	71
	References	72
	Curriculum Vitae	81

List of Figures

2.1	The solution and its approximation using nonharmonic system	21
4.1	Sample N points on the circle of radius r (a).	53
4.2	Sample N points on the circle of radius r (b).	53
4.3	Sample N points on the circle of radius r (c).	53
4.4	The eigenvalues distribution when $N=40$ and $N=60$	56
4.5	The eigenvalues distribution when $N=80$ and $N=100$	57
4.6	Singular values distribution, $N_1=40$, $N_2=50$, $M=400$	60
5.1	Basis Pursuit Recovery of Example 1	64
5.2	Compressed Sensing Recovery of Example 1	64
5.3	Basis Pursuit Recovery of Example 2	65
5.4	Compressed Sensing Recovery of Example 2	65
5.5	Frequency responses of original system, 5th approximation by Basis Pursuit and AFD for example 1 in noise and non-noise cases, respectively.	68
5.6	Frequency responses of original system, 5th approximation by Basis Pursuit and AFD for example 2 in noise and non-noise cases, respectively.	69

List of Tables

2.1	λ_n and c_n	20
5.1	List of a_k s and c_k s for example 1 in noise and non-noise cases, respectively.	68
5.2	List of a_k s and c_k s for example 2 in noise and non-noise cases, respectively.	69



