Uncertainty principles for quaternionic linear canonical transform and applications

by

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Abstract

In terms of this influential but rarely discussed theory, this thesis seeks to explore and generalize the linear canonical transform (LCT) to quaternion-valued signals. we call it the quaternionic linear canonical transform (QLCT).

In a communication theory setting, an uncertainty principle states that a signal cannot be arbitrarily confined in both the spatial and frequency domains. Many efforts have been devoted to extend the uncertainty principle to various types of functions and Linear canonical transforms. We establish an uncertainty principle for the QLCT by using the properties of the LCT and describe a lower bound on the product of the effective widths of quaternion-valued signals in the spatial and frequency domains by applying the uncertainty principle established in the first part, pointing out that only a Gaussian quaternionic signal can minimize the uncertainty.

One of the basic problems encountered in signal representations using the conventional LCT is the ineffectiveness of the LCT kernel to represent and compute location information. One method to overcome such a problem is the windowed Linear canonical transform (WLCT). Following this method we define windowed quaternionic linear canonical transform(WLCT). The QWLCT has the similar properties with QLCT. Finally we established uncertainty principle of QWLCT.

Contents

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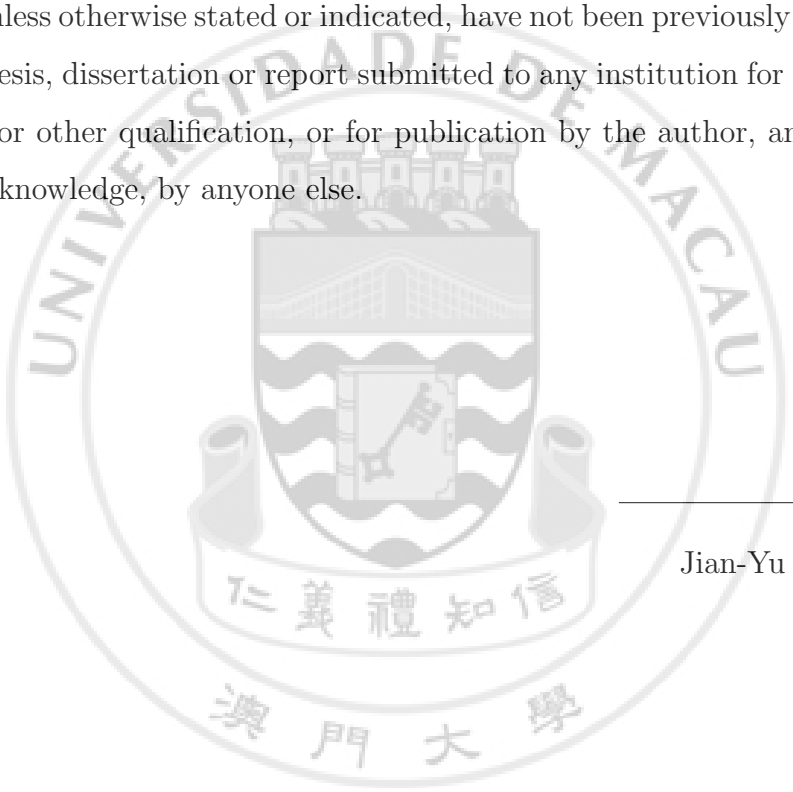
DECLARATION

The author declares that this thesis represents her own work with Dr. Kit-Ian Kou, the author's supervisor. All the work is done under the supervision of Dr. Kit-Ian Kou during the period 2011-2013 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.



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