Paley Wiener Theorem under Linear Canonical Transform of Slice Monogenic Functions

by

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Abstract

The classical Paley-Wiener theorem under Fourier transform has been found and proved for many years. The theorem is named for Raymond Paley (1907 to 1933) and Norbert Wiener (1894 to 1964). Later on, scholars extended this theorem to other integral transforms, like Fresnel transform, Jacobi transform, Dunkl Transform, etc. However, the Paley-Wiener Theorem under the linear canonical transform (LCT) also has been studied, like in [19]. The linear canonical transform is considered as a generalization of the traditional Fourier transform (FT), which has many applications to signal processing and optics [23, 24, 34]. It was first introduced in 1970s [22, 6] and is a four-parameter class linear integral transform. The LCT is also called the affine Fourier transform [2], the ABCD transform [4] and the generalized Fresnel transform [16]. Many operations, such as the FT, the fraction Fourier transform (FRFT) [23, 34], the Fresnel transform (FRT) [13], the Lorentz

transform [2] and scaling operations are its special cases. With more degrees of freedom compared to the FT and the FRFT, the LCT, as a powerful tool, has found many applications in filter design, signal synthesis, optics, radar analysis and pattern recognition, etc [23, 24]. The above mentioned applications indicate the great potential of LCT in signal processing. For instance, filtering in the LCT domain, proposed as in [3], can achieve better performance than in the FRFT domain because of more degrees of freedom. Especially when multi-component chirp signals interfere with the desired signal, only one filter is used in the LCT domain usually, but several filters are required in the FRFT domain [10].

Moreover, the higher dimensional extensions of Paley-Wiener theorem also have been studied for many years, for example, in [32], [31], [25], [5], [17]and [29], which were done under the Fourier transform. Particularly in [17], K. Kou and T. Qian gave a complete proof of the Paley-Wiener Theorem in R^n with the Clifford Analysis Setting, which is a brand new method.

Over the past thirty years, monogenic functions with Clifford values $\mathbb{R}_n$ have been studied successfully and widely. However, the identity function or the powers of the variable considered are not monogenic functions. In other words, an analytic function in complex variable may not be monogenic if we change the complex variable into Clifford variable directly. For more details, we can refer to [11, 8]. With this reason, scholars found a concept called slice monogenic functions with values in Clifford algebra, which is similar with the idea of analytic functions in complex analysis. With the concept of slice monogenic functions, scholars found many results, for example, [11] shows that monogenic functions can be related to power series and gives a complete proof of Cauchy integral formula as well as some of its consequences in Clifford algebra.

In this thesis, we first give some preliminaries in Chapter 1 that will be used in the following chapters, like the Fourier transform and its properties, the linear canonical transform and its properties, monogenic functions and slice monogenic functions, etc. In Chapter 2, we give a complete proof of Paley Wiener theorem under linear canonical transform with complex variable and the application of Paley Wiener theorem. The idea of this proof (of 1D Paley-Wiener theorem under linear canonical transform) is obtained from [19] for the window linear canonical transform. We extend the Paley Wiener theorem under linear canonical transform to slice monogenic functions in Chapter 3. Chapter 4 introduces monogenic kernel and gives a brief description about Paley Wiener theorem under quaternion linear canonical transform, which is a conjecture and can be a future work of our research. Finally, we do a conclusion as the last chapter of our thesis.

Contents

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DECLARATION

The author declares that this thesis represents her own work with Dr. Kit-Ian Kou, the author's supervisor. All the work is done under the supervision of Dr. Kit-Ian Kou during the period 2011-2013 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.

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