Investigation on the Unsymmetrical Probabilistic Solutions of Some Nonlinear Stochastic Oscillators with EPC Method

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Doctor of Philosophy in Civil Engineering

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Abstract

The nonlinear stochastic oscillators with unsymmetrical probabilistic solutions have not been discussed explicitly. In this research work, the analysis of the unsymmetrical probabilistic solutions of nonlinear stochastic oscillators is divided into two parts. One part about is about the nonlinear stochastic oscillators with even nonlinear term and another part is about the nonlinear stochastic oscillators with correlated excitations. Both even nonlinear term and correlated excitations can produce unsymmetrical probabilistic solutions.

The emphasis is on the investigation of the effectiveness of the exponential polynomial closure (EPC) method for obtaining the approximate stationary probabilistic solutions of the reduced Fokker-Planck-Komogorov (FPK) equations when the nonlinear stochastic oscillators contain even nonlinear term or when the external excitation and multiplicative excitation being Gaussian white noises are correlated.

The EPC method is extended to analyze the random oscillators with even nonlinear terms. Examples of different types and different nonlinear degrees of the even nonlinear term are investigated explicitly with EPC solution procedure. Compared with the results gained from Monte Carlo simulation (MCS) method, the results obtained by the EPC method with n=6 agree well with the simulated ones, especially in the tail region of the PDFs. Relative errors between approximate methods and numerical method are computed to show the effectiveness of the EPC method. Moments of different orders are calculated to show the influences of the magnitude and the sign of even nonlinearity on system responses.

Solution procedures are formulated for different oscillators for the approximate solution of the FPK equations. Different nonlinearity and correlated additive and multiplicative excitations are considered. Relative errors between approximate methods and numerical simulation are also analyzed. Compared with the simulated results, good agreement is achieved with the presented solution pro-

cedure, especially in the tail regions of the PDFs of system response. Response moments are obtained when the excitations are correlated or independent, respectively. Compared with the moments of the responses corresponding to independent excitations, nonzero means and unsymmetrical PDFs of system responses are presented when the additive and multiplicative excitations are correlated.

Declaration

I declare that the thesis here submitted is original except for the source materials explicitly acknowledged and that this thesis as a whole, or any part of this thesis has not been previously submitted for the same degree or for a different degree.

I also acknowledged that I have read and understood the Rules on Handling Student Academic Dishonesty and the Regulations of the Student Discipline of the University of Macau.

Table of Contents

List of Tables

List of Figures