

**Some New Results on the Hermite-Hadamard
Inequality for Functions of Two Variables
and Their Applications**

by

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A thesis submitted in partial fulfillment of the
requirements for the degree of

Master of Science in Mathematics

Faculty of Science and Technology

University of Macau

2013

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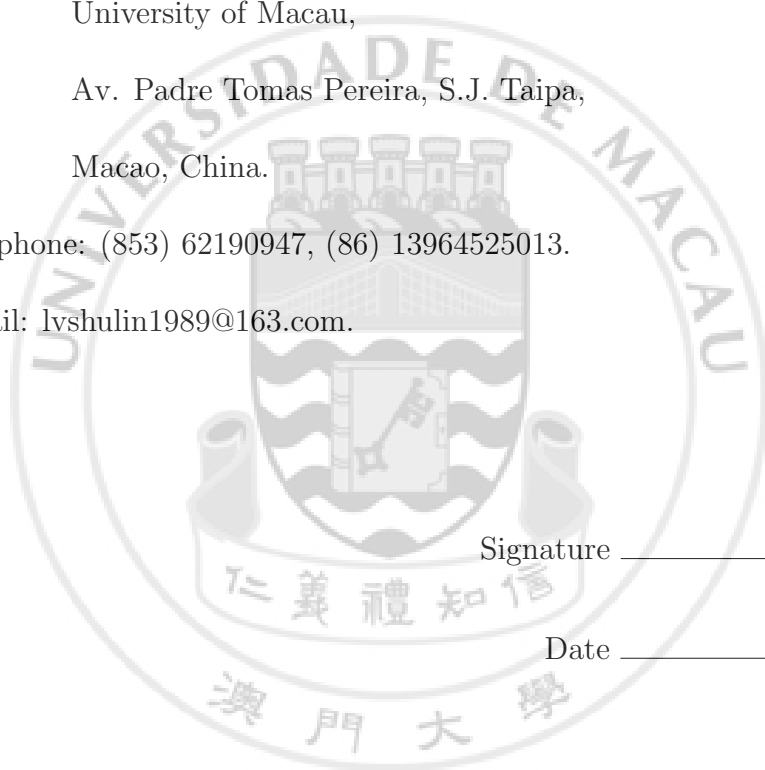
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ACKNOWLEDGEMENT

First and foremost, I wish to express my sincere and deep gratitude to my supervisor, Prof. Xiao-Qing JIN, for his support and inspiring guidance throughout my master's study. I thank him for showing me the beauty of scientific computing and teaching me the art of writing research papers. I thank him for his support on many research-related matters and his encouraging advice on my research.

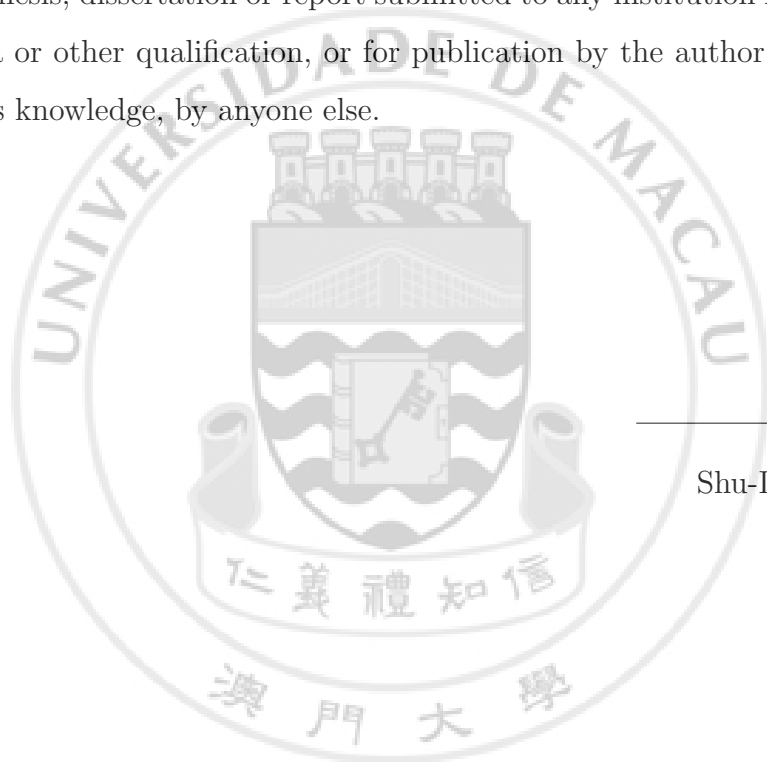
I would also deeply acknowledge other professors who have taught me courses during my master's study, including Prof. Yang CHEN, Prof. Tao QIAN, Prof. Che-Man CHENG, Prof. Deng DING, Prof. Hai-Wei SUN, Prof. Sik-Chung TAM, Prof. Seak-Weng VONG, Dr. Siu-Long LEI, Dr. Kit-Ian Kou, Dr. Hong-Kui PANG, etc. I have really benefited quite a lot on their classes and also out of class by communicating with them.

A special word of thanks is given to my dear friends, officemates and colleagues who have shared the wonderful years at the University of Macau: Mr. Qing-Jiang MENG, Mr. Zhi ZHAO, Mr. Zhi-Bo WANG, Mr. Zhi LI, Mr. Wei QU, Mr. Jian-Hao LV, Ms. Jia-Yu ZHENG, Ms. Yuan GAO, Ms. Hai-Lin ZHAO, Ms. Wen-Yan JIA, etc.

Finally, I want to thank my family for their endless love and support, which enables me to endure the hardships throughout these years.

DECLARATION

The author declares that this thesis represents her own work with Professor Xiao-Qing JIN, the author's supervisor. All the work is done under the supervision of Prof. Jin during the period 2011–2013 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.



Shu-Lin LV

To

My Parents



Some New Results on the Hermite-Hadamard Inequality for Functions of Two Variables and Their Applications

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Abstract

In this thesis, we consider the Hermite-Hadamard inequality for convex functions of two variables (HH2). In Chapter 1, a brief introduction of HH2 is given. In Chapter 2, we give two inequalities related to HH2, which are inspired by a result established recently. We apply them to two convex functions. Some results related to the p -logarithmic mean and the arithmetic mean are obtained. Applications of the classical Hermite-Hadamard inequality (for functions of one variable) to matrix functions are also given in this chapter. The thesis ends with a chapter of a brief conclusion.

Contents

1	Introduction	1
1.1	The Hermite-Hadamard inequality	2
1.2	Notations	3
2	Some New Results on HH2	5
2.1	Main results	5
2.2	Applications	7
2.2.1	Applications to special functions	7
2.2.2	Applications of the classical Hermite-Hadamard inequality to $f(tA)$	15
3	Conclusion	19
	Bibliography	21