Numerical ranges and spectrum of product of matrices

by

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Abstract

Let A be any $n \times n$ complex matrix. Let $\sigma(A)$, $W(A)$, and $W'(A)$ denote respectively the spectrum, the numerical range and the angular numerical range of A. For product of matrices, there are many inclusion results between the three sets. In particular,

(i) if A is a multiple of a positive semidefinite matrix, then $\sigma(AB) \subseteq W(A)W(B)$ for all $n \times n$ complex matrices B;

(ii) if $0 \notin W(A)$, then $\sigma(AB) \subseteq W'(A)W'(B)$ for all $n \times n$ complex matrices B.

In this thesis, we consider the converse of these results. For any fixed A, we find the necessary and sufficient conditions for the above inclusions to hold for all $n \times n$ complex matrix B. Some other inclusion relations between numerical range and angular numerical range are also considered.

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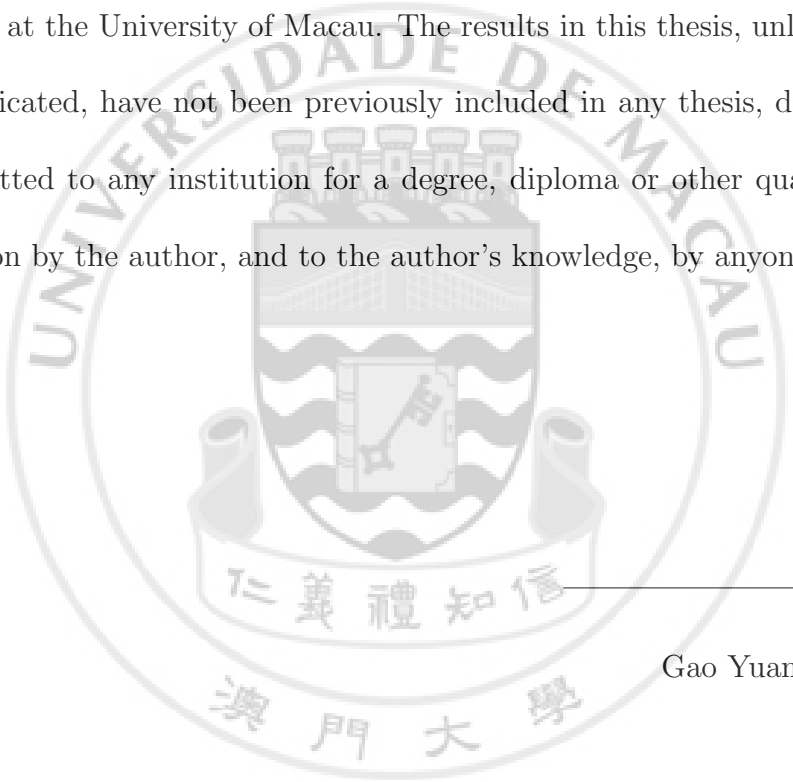
DECLARATION

The author declares that this thesis represents her own work with Professor Cheng Che Man, the author's supervisor. All the work is done under the supervision of Prof. Cheng during the period 2011-2013 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.



Gao Yuan

To

My Family