

Statistical Process Control Charts and Their Applications in  
Macao Schools

by

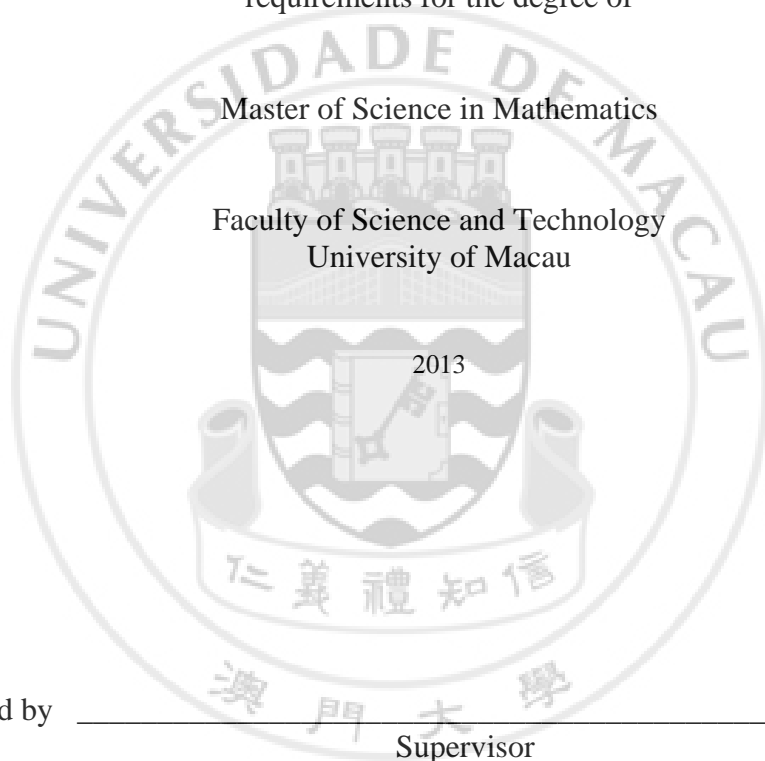
Ka Lei CHE

A thesis submitted in partial fulfillment of the  
requirements for the degree of

Master of Science in Mathematics

Faculty of Science and Technology  
University of Macau

2013



Approved by \_\_\_\_\_  
Supervisor

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Date \_\_\_\_\_

In presenting this thesis in partial fulfillment of the requirements for a Master's degree at the University of Macau, I agree that the Library and the Faculty of Science and Technology shall make its copies freely available for inspection. However, reproduction of this thesis for any purposes or by any means shall not be allowed without my written permission. Authorization is sought by contacting the author at

Address: Faculty of Science and Technology,  
University of Macau,  
Av. Padre Tomás Pereira, S.J. Taipa,  
Macau, China.

Telephone: (853) 66305063

E-mail: kelly\_che1230@yahoo.com.hk

Signature \_\_\_\_\_

Date \_\_\_\_\_



University of Macau

Abstract

## **Statistical Process Control Charts and Their Applications in Macao Schools**

by Ka Lei CHE

Thesis Supervisor: Professor Deng DING  
Department of Mathematics

Statistical Process Control (SPC) enables the use of one or more control charts to control process statistically and prevent quality problems without delay. The traditional control charts were first proposed by Walter A. Shewhart in 1924 to determine the stability of process parameters. This is based on collecting data from the process in order to estimate the model parameters. Control limits are determined from these estimates and used to monitor future data. Bayesian statistics combines prior knowledge and likelihood function via Bayesian theorem to predict the unknown parameters. Recently, Bayesian methods are widely used in SPC for application cases of small sample sizes.

Classical process control methods are not suitable when the number of data is small and, during a base period, do not have control limits. In such cases, it is almost impossible to calculate the limits and have a control chart during the base period. Thus, we use Bayesian method to compute the posterior probability based on historical and latest data to judge process stability. In the case of small sample sizes, the Bayesian method in statistical process control yields more efficient results.

In this thesis, various statistical process control methods are introduced, such as several traditional Shewhart control charts:  $p$ ,  $np$ ,  $c$ ,  $u$ ,  $\bar{X}$  &  $R$ ,  $\bar{X}$  &  $S$ ,  $I$  &  $MR$  charts, etc. By using Bayesian method, several improved process control charts are studied, and compared with traditional control charts. These improved process control charts

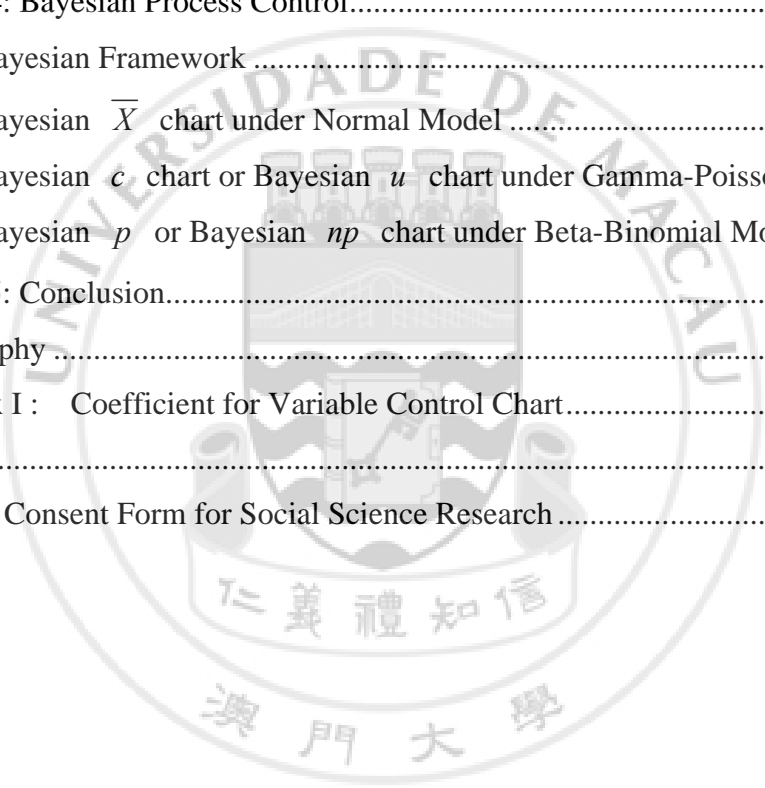
have more reliability and smaller probabilities of error alarm, which are caused by traditional control charts under small sample sizes. Some of these process control charts are applied to monitor changes in Macao high school. Numerical results of these applications show that process control charts have important applications in high school management and quality control.



# Table of Contents

List of figures.....	iii
List of tables.....	v
Chapter 1: Introduction.....	1
1.1 An Introduction to Statistical Process Control.....	1
1.2 Some Approaches to Statistical Process Control.....	3
1.2.1 The Frequentist Approach to Statistical Process Control.....	3
1.2.2 The Bayesian Approach to Statistical Process Control.....	4
1.3 Organization of this thesis.....	5
1.3.1 Main parts of this thesis.....	5
1.3.2 Structure of this thesis.....	6
Chapter 2: Basic Concepts of Statistical Process Control.....	7
2.1 Tools of Statistical Process Control.....	7
2.2 Control Chart.....	8
2.2.1 Basic Principles.....	8
2.2.2 Control Limits and Errors.....	9
2.2.3 Sampling.....	10
2.2.4 Rational Subgroup.....	11
2.3 Control Charts and Hypothesis testing.....	12
2.4 Rules for Control Charts.....	12
2.5 The Types of Control Charts.....	15
Chapter 3: Shewhart Control Charts.....	17
3.1 $3\sigma$ Principle of Shewhart Control Charts.....	17
3.2 Phase I and Phase II of Control Chart Applications.....	18
3.3 Sample Size of Control Charts.....	19
3.4 Process Capability.....	20
3.5 OC Curves in Quality Control.....	22
3.6 Shewhart Control Charts.....	22

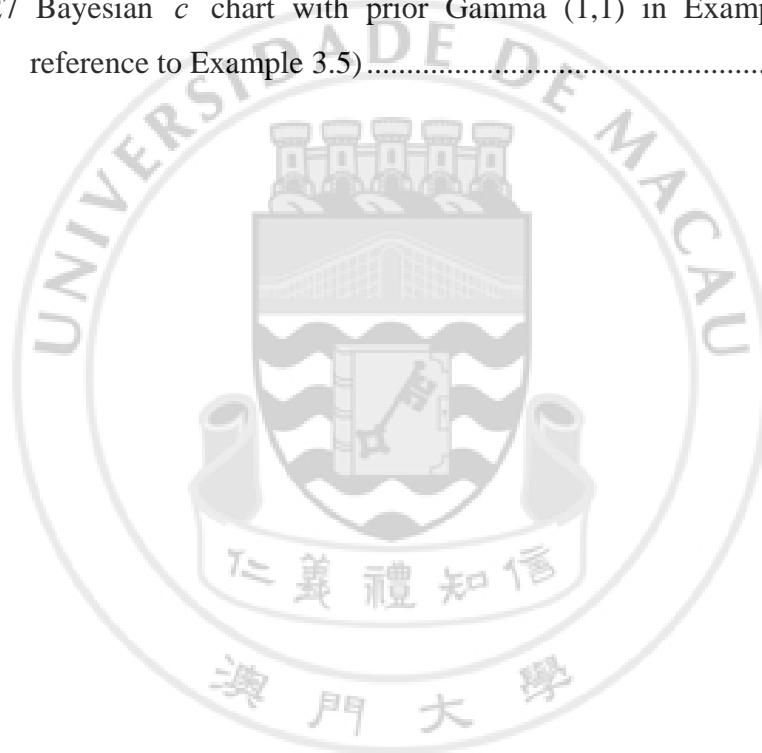
3.6.1 Variable Control Charts .....	22
3.6.1.1 $\bar{X}$ & $R$ Charts .....	22
3.6.1.2 $\bar{X}$ & $S$ Charts.....	26
3.6.2 $I$ & $MR$ charts.....	31
3.6.3 Attribute Control Charts .....	35
3.6.3.1 $c$ chart and $u$ chart .....	35
3.6.3.2 $p$ chart and $np$ chart .....	42
Chapter 4: Bayesian Process Control.....	47
4.1 Bayesian Framework .....	47
4.2 Bayesian $\bar{X}$ chart under Normal Model .....	49
4.3 Bayesian $c$ chart or Bayesian $u$ chart under Gamma-Poisson Model.....	53
4.4 Bayesian $p$ or Bayesian $np$ chart under Beta-Binomial Model .....	64
Chapter 5: Conclusion.....	68
Bibliography .....	70
Appendix I: Coefficient for Variable Control Chart.....	75
Vita.....	76
Informed Consent Form for Social Science Research .....	77



# List of figures

<i>Number</i>	<i>Page</i>
Figure 1 A typical control chart .....	9
Figure 2 Type I error in control chart .....	10
Figure 3 Rule 1: Any point beyond Zone A .....	12
Figure 4 Rule 2: Two out of three consecutive points fall within Zone A or beyond .....	13
Figure 5 Rule 3: Four out of five consecutive points fall within Zone B or beyond .....	13
Figure 6 Rule 4: Eight consecutive points fall on the same side of center line .....	14
Figure 7 Types of control charts .....	16
Figure 8 The flow of establishing the control chart .....	19
Figure 9 Process capability of 1.33 – process not centered .....	21
Figure 10 $\bar{X}$ & $R$ charts in Example 3.1 .....	25
Figure 11 $\bar{X}$ & $S$ charts in Example 3.2 .....	29
Figure 12 $I$ & $MR$ charts in Example 3.3 .....	33
Figure 13 Normal probability plot of resistivity in Example 3.3 .....	33
Figure 14 $c$ chart in Example 3.4 .....	36
Figure 15 OC curve of $c$ chart in Example 3.4 .....	38
Figure 16 $u$ chart in Example 3.5 .....	40
Figure 17 $np$ chart in Example 3.6 .....	44
Figure 18 OC curve of $np$ chart in Example 3.6 .....	46
Figure 19 Shewhart and Bayesian control for $\bar{X}$ chart in Example 4.1 .....	51
Figure 20 Bayesian $u$ chart with prior Gamma (16,16) in Example 4.2 (With reference to Example 3.4) .....	58
Figure 21 Bayesian $u$ chart with prior Gamma (10,10) in Example 4.2 (With reference to Example 3.4) .....	58
Figure 22 Bayesian $u$ chart with prior Gamma (5,5) in Example 4.2 (With reference to Example 3.4) .....	59

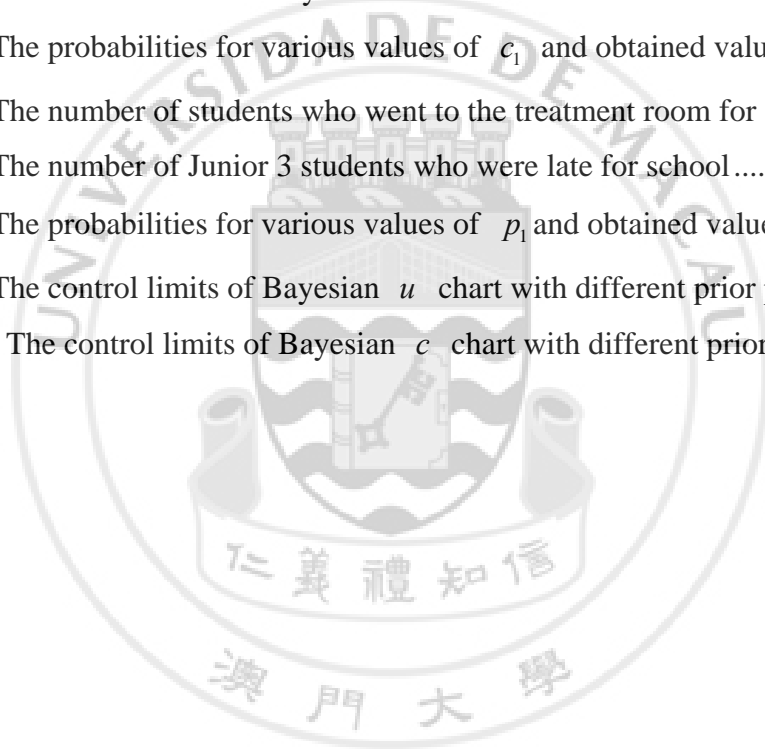
Figure 23 Bayesian $u$ chart with prior Gamma (1,1) in Example 4.2 (With reference to Example 3.4).....	59
Figure 24 Bayesian $c$ chart with prior Gamma (16,16) in Example 4.2 (With reference to Example 3.5).....	61
Figure 25 Bayesian $c$ chart with prior Gamma (10,10) in Example 4.2 (With reference to Example 3.5).....	61
Figure 26 Bayesian $c$ chart with prior Gamma (5,5) in Example 4.2 (With reference to Example 3.5).....	62
Figure 27 Bayesian $c$ chart with prior Gamma (1,1) in Example 4.2 (With reference to Example 3.5).....	62





# List of tables

<i>Number</i>	<i>Page</i>
Table 1 The number of Junior students who forgot to bring their textbooks .....	26
Table 2 The number of Junior students who were late for assignment submission .....	30
Table 3 The number of secondary students who borrowed books from the library .....	34
Table 4 The number of secondary students who went to the treatment room .....	37
Table 5 The probabilities for various values of $c_1$ and obtained values for $\beta$ .....	39
Table 6 The number of students who went to the treatment room for stomach pain .....	41
Table 7 The number of Junior 3 students who were late for school .....	45
Table 8 The probabilities for various values of $p_1$ and obtained values for $\beta$ .....	46
Table 9 The control limits of Bayesian $u$ chart with different prior parameters .....	57
Table 10 The control limits of Bayesian $c$ chart with different prior parameters.....	60



# Acknowledgments

First and foremost, I would like to express my deepest gratitude to my supervisor, Professor Deng DING. Thanks to his support, encouragement and advice, I have been able to complete this thesis. In the last two years, his patient guidance and academic supervision have brought me tremendous personal growth. I can say in all honesty that I have been positively affected by his good attitude.

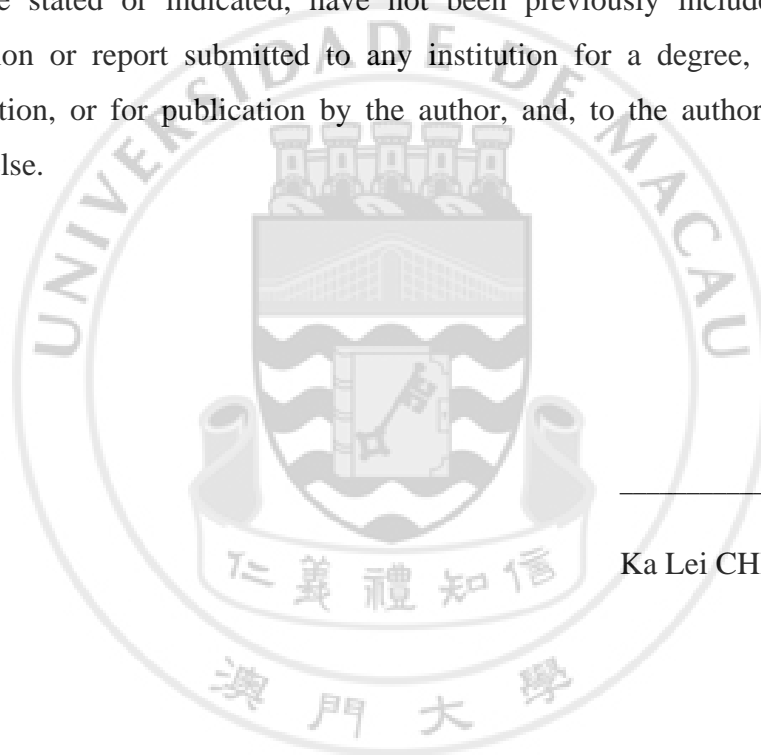
I am also grateful to my other teachers in the Department of Mathematics for their valuable advice and assistance during my M.Sc. studies. Among them, I would especially like to thank Professor Tao QIAN, Professor Che Man CHENG and Professor Sik Chung TAM.

I would like to thank my colleagues at the Santa Rosa de Lima (Chinese Section), especially the former principal Sister Chui King LO, for her tremendous support, tolerance and understanding. I am grateful that she afforded me a very considerate adjustment to my work schedule, so I could concentrate better on my studies. I also wish to extend a note of appreciation to the new principal Mr. Chi Chau IEONG, for his advice and permission to use the school's data. I am equally indebted to my colleagues in Santa Rosa de Lima for their help, especially in reducing my work pressure during this period. My gratitude goes to Ms. M. I. Ip, Ms. S. W. Chio, Ms. S.N. Cheang, Ms. C.H. Wong, Ms. Z.Z. Yang, Ms. S.M. Tsang and Ms. S.L. Lei.

I have also benefited from the time spent with colleagues at the University of Macau. My gratitude also goes to Mr. K.H. Chan, for his sincere help and generous advice during this period; Ms. F.S. Tong, Mr. Y.T. Ho, Mr. W. Mei and Jack. I would like to thank my teachers, friends and students. They helped and encouraged me throughout. A note of gratitude also goes to Mr. M.S. Ip, Peter, Annie Lei, Vina Lei, Kathy Lou, Ida Cheang, Miki Che, Tracy Kuok, Mui Chan, Usf, Kailyn Mou, Karen Cheong and Emmanuel, et al. Finally, I am grateful to my parents for their eternal support and endless love.

# Dedication

The author declares that this thesis represents her own work with Professor Deng DING, the author's supervisor. All the work is done under the supervision of Professor Deng DING during the period 2011-2013 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and, to the author's knowledge, by anyone else.



---

Ka Lei CHE

To

My Parents

