# A Splitting Iteration Method for Solving Fractional Diffusion Equations

**by**

**Wei QU**

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**Faculty of Science and Technology**

**University of Macau**

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A thesis submitted in partial fulfillment of the
requirements for the degree of

Master of Science in Mathematics

Faculty of Science and Technology
University of Macau

2013

Approved by _____
Supervisor

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Date _____

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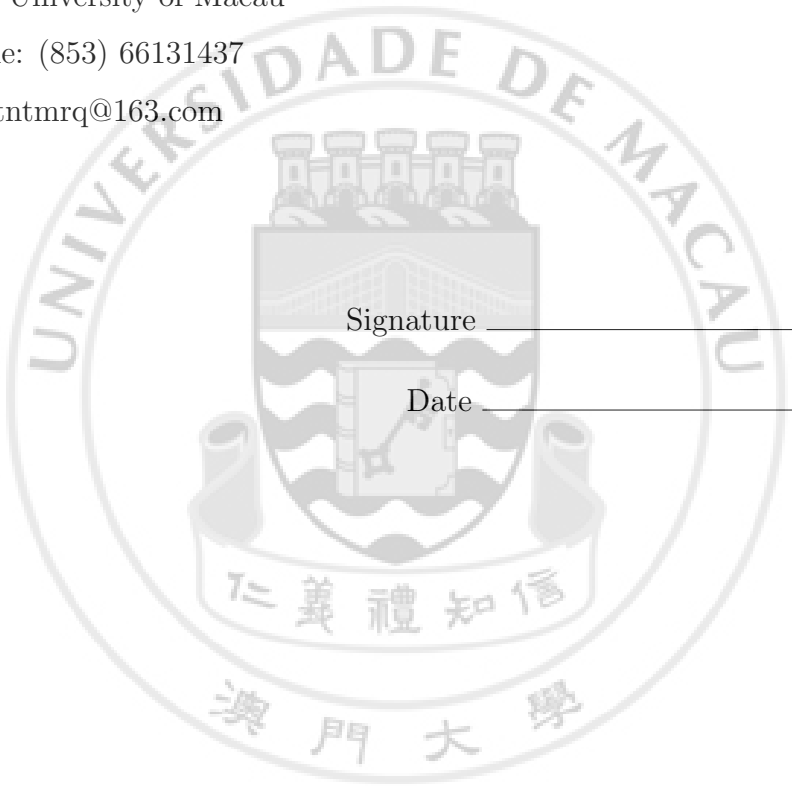


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# DECLARATION

The author declares that this thesis represents his own work based on the ideas suggested by Dr. Siu-Long Lei, the author's supervisor. All the work is done under the supervision of Dr. Lei during the period 2011 – 2013 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.

QU WEI

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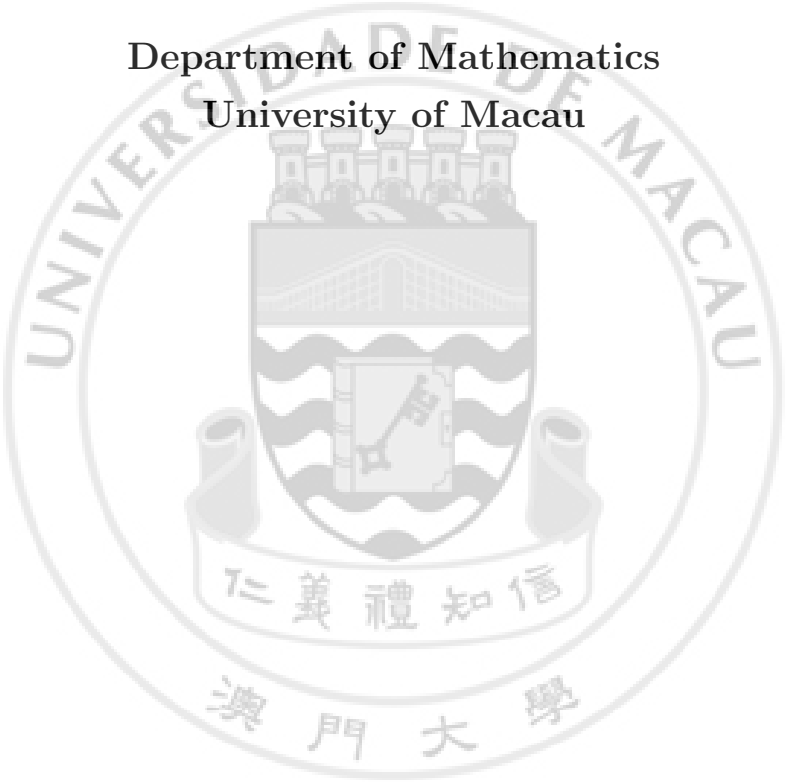

# Abstract

In this thesis, we solve the two-sided space fractional diffusion equations (SFDEs) associated with an initial condition and zero Dirichlet boundary conditions. We use the Crank-Nicolson (C-N) technique with a second order discretization fractional operators for solving the two-sided SFDEs, and we prove that the scheme is unconditionally stable and second order in time and space. Moreover, we prove the circulant and skew-circulant splitting (CSCS) iteration method is employed to solve the unsymmetric Toeplitz system, which is discretized from SFDEs problem effectively. We also prove that the two splitting (circulant and skew-circulant) matrices are positive stable for any choice of time step and space step, so that the CSCS iteration method converges to the unique solution of the Toeplitz system. Numerical results are provided to verify the accuracy and efficiency of the CSCS iteration method.

In addition, a second order finite difference scheme for two-sided SFDEs problem (1.1) with variable diffusion coefficients is also studied in this thesis. The scheme is proved to be unconditionally stable and convergent if $\alpha \in (\alpha_0, 2)$ where $\alpha_0 \approx 1.5545$. Numerical examples are shown to support the theoretical results.

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