

**SHEAR CORRECTION FACTOR FOR FGM
PLATE BASED ON THICKNESS SHEAR
VIBRATION**

by

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Abstract

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Thesis Supervisor: Prof. Iu Vai Pan

Functionally graded materials have continuous variation of material properties from one surface to another, which is different from the stepped or discontinuous composite material. The gradation of properties in FGM reduces the thermal stresses, residual stresses, and stress concentrations found in traditional composites. Due to the high quality of the FGM, it meets the requirement of the growing new technologies especially in aerospace, microelectronics, biomechanical, nuclear engineering and so on.

The first order shear deformation theory (FSDT) is a high efficient and convenient method for analyzing the moderate thick plates. But the shear correction factor of plate is a necessary parameter in FSDT to cater the non-uniformed shear strain in transverse section. The shear correction factor of homogenous plate is not suitable for the FGM plate, as the variation of material properties is not considered in the theory. This thesis gives a convenient and effective method to obtain the shear correction factor for FGM plates based on thickness shear vibration. Thickness shear vibration is an in-plane vibration of infinite plates characterized by shear deformation. The frequencies of thickness shear vibration of an infinite FGM plate according to three-dimensional theory of elasticity is determined by Galerkin method. The shear correction factor can then be obtained by matching the fundamental frequencies of thickness shear vibration obtained by three-dimensional theory of elasticity and by FSDT.

Parametric studies of the shear correction factor with the variation of material properties, such as the mass density ratio, the Young's modulus ratio and the material index are discussed. Both the FGM plates and sandwich FGM plates are considered. The results show that the shear correction factor for FGM plates with the material index larger than 1

is small than $\pi^2/12$ which is the shear correction factor for homogeneous plates. The comparisons of the shear correction factor from presented method and from the other method are also presented. The relative difference of shear correction factor is increased when the Young's modulus ratio R_y increase. It is observed that the Young's modulus ratios R_y have bigger influence on the shear correction factor in the thickness shear vibration method. The relative difference of shear correction factor is decreased when the mass density ratio R_m increase, it means that the relative difference of shear correction factor will be very small when the proper value of R_m has been used. Using the present shear correction factor and the first-order shear deformation theory, the free vibration frequency of simply-supported rectangular FGM plates is determined by the Navier's solution method. Comparison studies are given for the frequency parameters obtained by different shear correct factors and by different numerical model.

Key words: FGM plate, shear correction factor, thickness shear vibration, first order shear deformation theory.

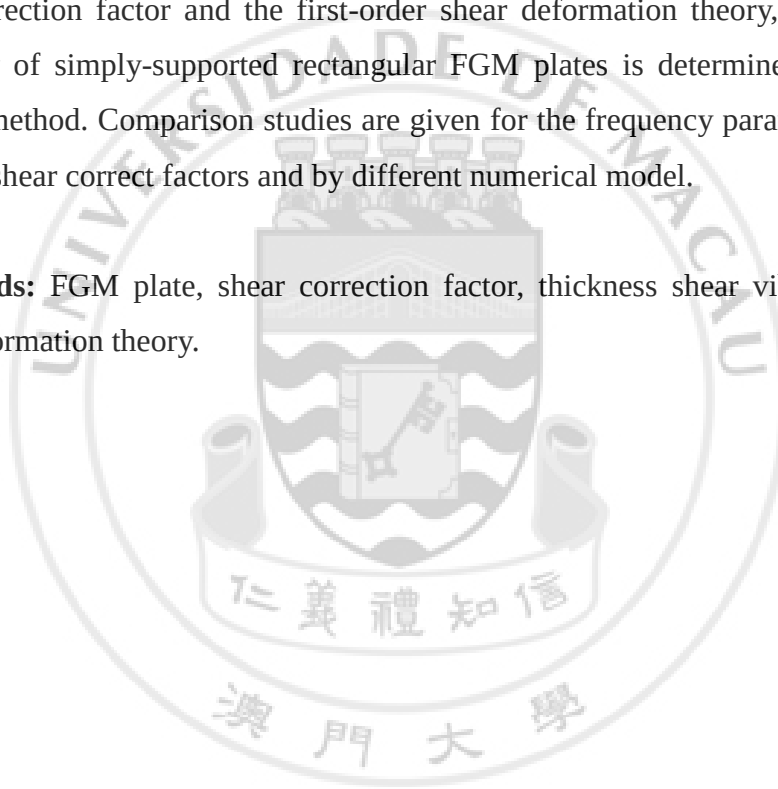


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Nomenclature

a	The length of plate
b	The width of plate
g	Power law function
h	The thickness of plate
k	Shear correction factor
p	Material index
t	Time
u_i	displacement
u, v, w	Mid-plane displacement
$\tilde{u}, \tilde{v}, \tilde{w}$	Non-dimensional mid-plane displacement
x_i	Rectangular coordinate axis
A, B, D	Coefficient related to material properties
$\bar{A}, \bar{B}, \bar{D}$	Non-dimensional coefficient related to material properties
E	Young's modulus
G	Shear modulus
$\bar{G}$	Non-dimensional shear modulus
I_1, I_2, I_3	The inertia moment
$\bar{I}_1, \bar{I}_2, \bar{I}_3$	Non-dimensional inertia moment
N_1, N_2, N_{12}	The force
$\tilde{N}_1, \tilde{N}_2, \tilde{N}_{12}$	The non-dimensional force
M_1, M_2, M_{12}	The bending moment
$\tilde{M}_1, \tilde{M}_2, \tilde{M}_{12}$	The non-dimensional bending moment
$\mathbf{K}, \mathbf{M}, \mathbf{U}$	Coefficient matrix
P, P_t, P_b	Material properties
Q_1, Q_2	The shear force
$\tilde{Q}_1, \tilde{Q}_2$	The non-dimensional shear force
R_y	Young's modulus ratio
R_m	Mass density ratio

R_t	the layer ratios
$U_{ij}, V_{ij}, W_{ij}, M_{ij}, N_{ij}$	Constant coefficient
Z	Shape functions
$\bar{Z}$	Non-dimensional shape functions
ε_{ij}	Strains
σ_{ij}	stress
ν	Poisson's Ratio
ρ	Mass density
$\bar{\rho}$	Non-dimensional mass density
ω	Frequency of plate
ψ_1, ψ_2	Rotation
$\tilde{\psi}_1, \tilde{\psi}_2$	Non-dimensional rotation
β	Frequency parameter
λ	Frequency parameter
η	The ratio of length and width
τ	The ratio of length and thickness

Indices

i, j	1, 2, 3, ...
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