DUAL BOUNDARY ELEMENT ANALYSIS OF FATIGUE CRACK GROWTH IN TWO-DIMENSIONAL LINEAR ELASTIC FRACTURE PROBLEM

by

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Abstract

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By LI JIAN TAO

Thesis Supervisor: Professor Kou Kun Pang

ABSTRACT

The objective of this study was the analysis of fatigue crack growth in two-dimensional case and presents an analysis of mixed-mode fracture problems arising in isotropic material. Basic concepts of fracture mechanics are discussed to analyze their applicability in modeling fatigue crack propagation. Special attention was given to the calculation of stress intensity factors of crack tips under mixed Mode I and Mode II in linear elastic fracture problems.

Boundary element method and the J integral used in this thesis were accurate techniques widely used to the computation of stress intensity factors K_I and K_{II}. The recent developed dual boundary element method was introduced to analysis the fatigue crack growth. The basic equations of this method were the displacement and the traction boundary integral equations (BIEs). Applying the first BIE on one of the crack surfaces and the second one on the other, one can solve the general crack

problems in a single-region analysis. Finite element based Franc2D is also used in the fracture analysis to verify the results of the boundary element method.

Examples of geometries with an embedded eccentric crack and two inclined cracks were analyzed using dual BEM and FEM. The accuracy of stress intensity factors obtained from BEM was verified by those obtained from FEM and the analytical ones in relative reference. It was found in this work that the dual boundary element method was an effective method in the analysis of fatigue crack extension. For the fatigue analysis, numerical examples of problems under pure mode I and mixed mode deformations were given. Another example was given to present the whole process of fatigue analysis for a specify crack problem.

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LIST OF SYMBOLS

Nomenclature

σ	[MPa]	mechanical stress
ε	[-]	engineering strain
v	[-]	Poisson's ratio
E	[Mpa]	modulus of elasticity
G	[Mpa]	shear modulus
K_I	[MPa $m^{1/2}$]	mode I stress intensity factor
K_{II}	[MPa $m^{1/2}$]	mode II stress intensity factor
J	[J/m^2]	J integral
K_{eff}	[MPa $m^{1/2}$]	effective stress intensity factor
ΔK_{eff}	[MPa $m^{1/2}$]	change in effective stress intensity factor
K_{max}	[MPa $m^{1/2}$]	maximum stress intensity factor
K_{min}	[MPa $m^{1/2}$]	minimum stress intensity factor