A Second Order ADI Method for 2D Parabolic Equations with Mixed Derivative

by

ZHANG Chun Yang

A thesis submitted in partial fulfillment of the
requirements for the degree of

Master of Science in Mathematics

Faculty of Science and Technology
University of Macau

2012

Approved by _____

Supervisor

_____

_____

_____

Date _____

In presenting this thesis in partial fulfillment of the requirements for a Master's degree at the University of Macau, I agree that the Library and the Faculty of Science and Technology shall make its copies freely available for inspection. However, reproduction of this thesis for any purposes or by any means shall not be allowed without my written permission. Authorization is sought by contacting the author at

Address: P.O. Box 3001 Macau

Telephone: (853)8397 8530

E-Mail: zhangchunyangg@gmail.com



Signature _____

Date _____

ACKNOWLEDGEMENT

I would like to express my sincere gratitude to my supervisor, Prof. Hai-Wei Sun, for his inspiring guidance, constant encouragement and help throughout my M.Sc. studies and in the preparation of this thesis.

I also would like to show my deeply appreciations to other teachers working in the Department of Mathematics, and staffs in the facilities for their warm-hearted assistance and support in my Master study. Among them are Prof. Xiao-Qing Jin, Prof. Tao Qian, Prof. Deng Ding, Prof. Che-Man Cheng, Prof. Sik-Chung Tam, Dr. Seak-Weng Vong, Dr. Kit-Ian Kou and Dr. Siu-Long Lei.

A special word of thanks is given to other friends who helped me and encouraged me with their friendship. They are my classmates Rong Shi, Wei-Xiong Mai, Si-Fan Wu and Jing-Ya Zhao.

At last, I would like to express my deep love and appreciation to my family for their support and understanding. They will always play as my backbone.

DECLARATION

The author declares that this thesis represents his own work based on the ideas suggested by Prof. Hai-Wei Sun, the author's supervisor. All the work is done under the supervision of Prof. Sun during the period 2010 – 2012 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.

ZHANG Chun Yang

UNIVERSIDADE DE MACAU
仁義禮知信
澳門大學

Contents

1 Preliminary **1**

1.1 Introduction 1

1.2 Problems 3

1.3 Alternating direction implicit method 4

1.4 Von Neuman stability analysis 7

1.4.1 Numerical stability 7

1.4.2 Illustration of the method 8

1.5 Stability of the ADI scheme 11

2 Crank-Nicolson method **13**

2.1 Brief introduction 13

2.2 Properties 15

3 Douglas-Rachford splitting method **17**

3.1 Operator splitting techniques 17

3.1.1 Operator splitting 17

3.1.2 Second-order operator splitting 18

3.2 Douglas scheme 19

3.3 Douglas-Rachford scheme 21

4 Presented ADI scheme **25**

4.1 Motivation 25

4.2 Skew-ADI scheme 27

5 Numerical results and concluding remarks **29**

5.1 Numerical experiments 29

5.1.1 Example 1 29

5.1.2 Example 2 31

5.2 Conclusions 33

Bibliography **35**