

Applications of Adaptive Fourier Decomposition to Financial
Data

by

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Introduction

There exist many kinds of methods for analyzing financial time series. The statistical methods of data collected sequentially in time should be originated from Bachelier, who proposed a Brownian motion model for stock price. Hereafter, there have been lots of developments. In the books [1], [2], the authors give a systematic account of linear time series models and their applications to modeling and prediction of data. But these methods are all based on the assumption that time series under study is stationary and linear. However, the assumption of stationarity and linearity is sometimes hard to hold for financial time series.

Afterwards, financial time series is gradually analyzed by methods which are often used in engineering area, including the Fourier method, the wavelet method and the empirical mode decomposition (EMD) method. These methods have advantages as well as disadvantages. The Fourier method has strict condition that time series must be stationary and linear. The wavelet method is parametric and more suitable to image processing. The EMD method does not have strict mathematical model and its further mathematical analysis is impossible.

A new method, adaptive Fourier decomposition (AFD), possesses a natural ability to decompose a time series into several subseries, just like the EMD method and the wavelet method. The key points are that the AFD method not only has strong theoretical foundation but also has no restrictions on data. In this thesis, We will give a summary of this method, and then apply it to analysis of a financial record from stock price. It is hoped that this new method, which has been found applicable in speech, might also be found some other unique and useful applications in the financial area.

A natural concept in financial time series is the notion of multi-scale features, since markets consist of agents working on different time horizons.

With AFD, we can at least approximately decompose a financial time series into subseries presenting contributions of different agents. Therefore, we are decomposing the financial time series to their natural components. Thus, we achieve better understanding of the dynamics of the financial data.

Specifically, two different applications in financial time series data are presented by AFD in this thesis. These applications aim to extend AFD to new research areas in finance. The two applications are as follows:

1. Approximation, trend extraction and filter of financial time series;
2. Hybrid AFD-neural network in prediction of financial time series.



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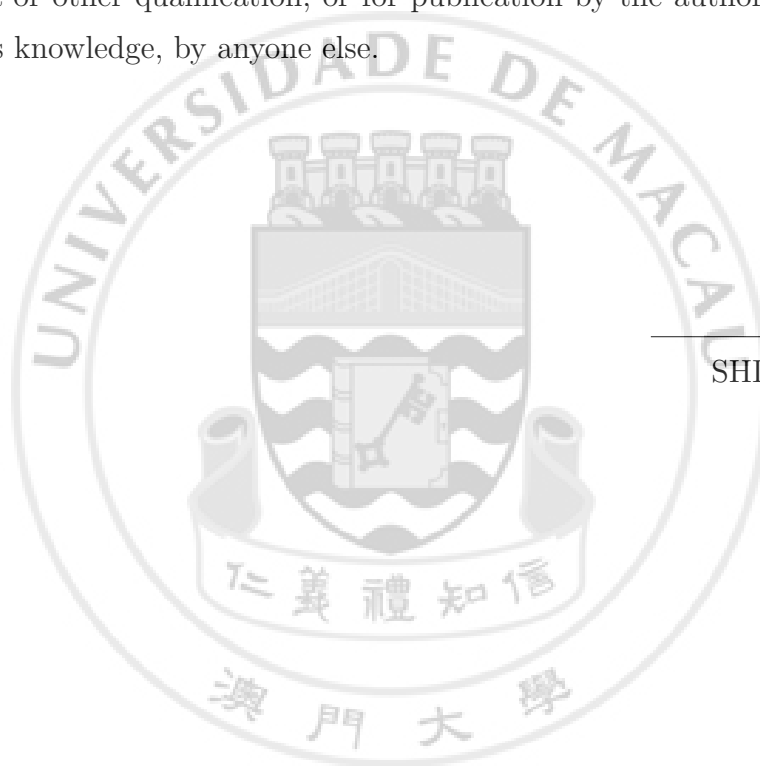
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DECLARATION

The author declares that this thesis represents her own work with Professor Tao Qian, the author's supervisor. All the work is done under the supervision of Professor Qian during the period 2010-2012 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.



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