A Revision of Adaptive Fourier Decomposition

by

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Introduction

An alternative method other than Fourier based ones for processing signal, the so called Adaptive Fourier Decomposition, is proposed by Prof. Tao Qian, having the merits of a compact expression and good analytic property. A lot of people was inspired by this work, and application were found in many fields, including noise deduction, system identification and speech analysis.

In this article, we generalize the method from the space H^2 to L^2, and prove that some important identities still hold in the new space. Experiments are carried out to support the generalization. We draw the conclusion and give some advice on the potential usage of the new approach.

In Chapter 1, a short introduction is given to Adaptive Fourier Decomposition and its generalization to L^2. Hilbert transform, as a byproduct of AFD, is also valid under this new circumstance.

In Chapter 2, we give the formal proof of the generalization and some identities. Remarks on simplification of the computation are given. We also proposed a new technique to deal with incomplete data and its application.

In Chapter 3, we shows the experiments. The first one is retrieving H^2 part of a signal in L^2. The second one is on the new technique. Analysis are supplied for each experiment.

In Chapter 4, we draw the conclusion based on our numerical results. The advantage and possible application of the generalization is summarized in detail.

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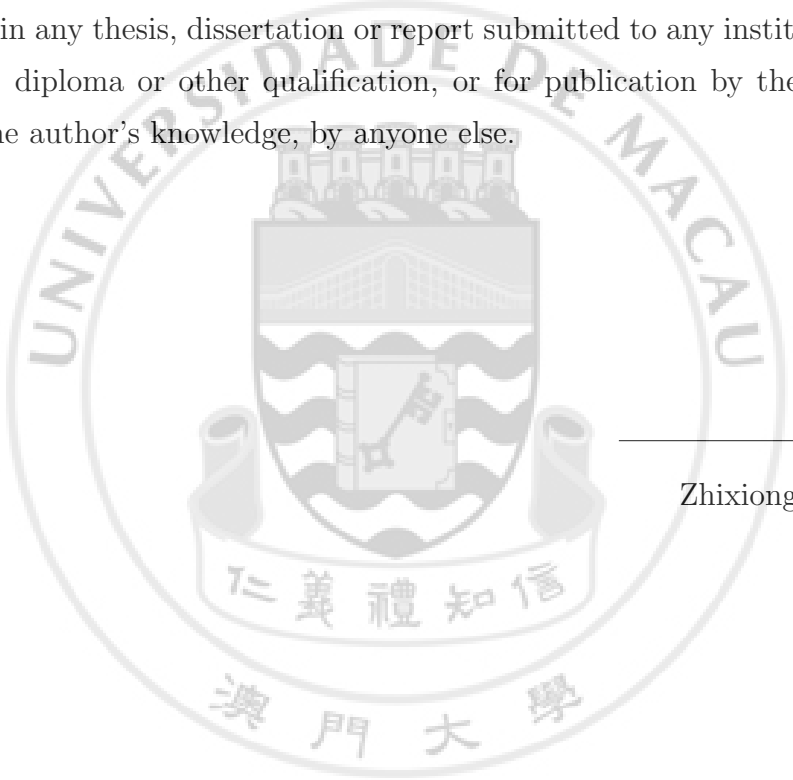
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DECLARATION

The author declares that this thesis represents his own work with Professor Tao Qian, the author's supervisor. All the work is done under the supervision of Professor Qian during the period 2009-2012 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.



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