Numerical Study of Stokes' Second Flow Problem

by

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Abstract

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The motion of a incompressible fluid induced by the sinusoidal oscillation of a infinite plane flat plate is termed Stokes' second flow problem. With the assumption of zero velocity normal to the plate and thus simplified Navier-Stokes equations, Stokes' well-known result can be obtained by analytical approach. In this thesis, behaviors of the fluid are to be studied by solving the full Navier-Stokes equations through numerical simulations. As the full Navier-Stokes equations are being solved numerically, existence of velocity normal to the oscillating plate, and even further to its behaviors, may be observed. Besides, the motion of fluid induced by the sinusoidal oscillation of a finite plane flat plate is also to be studied using moving mesh method since it is more practical. Compared to the Stokes' second flow problem, whose boundaries and mesh are stationary, the boundaries, which representing the finite plate in simulations of flow induced by a finite plane flat plate, are in motion during the whole simulation process. The geometry of the domain of the flow induced by a finite oscillating plate is changing from time to time due to the motion of the moving boundaries. This causes large deformation of mesh of the domain. The stationary mesh is no longer suitable when representation of reality of this problem and accurate results are wanted. Base on the mentioned reasons, to make the simulations of the flow induced by a finite

oscillating plate more closer to the reality, moving mesh method is used. The ratio of physical simulation time of using moving mesh method to that of using stationary mesh method is about 12 to 20, providing that the settings (geometry of the domain, boundary conditions, number of mesh elements, mesh distribution, etc.) of using the two different meshing methods are the same.

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LIST OF ABBREVIATIONS

CFD	**C**omputational **F**luid **D**ynamics
FEM	**F**inite **E**lement **M**ethod
FIOFP	**F**low **I**nduced by an **O**scillating **F**inite **P**late
FD	**F**inite **D**ifference
FV	**F**inite **V**olume
PDE	**P**artial **D**ifferential **E**quation
SSFP	**S**tokes' **S**econd **F**low **P**roblem

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