Kinematics, Dynamics and Control Analysis for Micro Positioning and Active Vibration Isolation Using Parallel Manipulators

by

Yuan Yun

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ABSTRACT

Nanotechnology requires a new field termed nanomanipulation to deal with how to handle components and structures in nanometer scale by utilizing devices with high positioning accuracy and dexterous motion of the end-effector and controlling external forces with sensory feedback, which has been enabled by the invention of bio-cell manipulation, optical fibers alignment, micro device assembly, and scanning probe microscopes. This thesis presents a general dynamics and vibration control method of a class of parallel or hybrid robots at first. The dynamic equations of a class of parallel/hybrid manipulators are developed using Kane's method. Secondly, this thesis presents a 3-PUU parallel platform by using the general dynamics and control method. A mixed H_2/H_∞ controller is obtained using LMI control toolbox of MATLAB. The results of the time history responses and selected frequency responses of the moving platform along each axis are given to show the isolation effect due to the good performance of designed controller.

Furthermore, given the advantages of parallel manipulators and compliant manipulators, a 3-PUPU parallel manipulator with flexure hinges at all joints has been developed to provide micro positioning manipulation in a large workspace and an active vibration isolation for future possible applications in tracking cells, cell injection, crystal growth experiment in specific acceleration level, and so on. The procedure of kinematics modeling of the macro parallel mechanism system is presented via the stiffness model and Newton-Raphson method because of adopting a kind of wide-range flexure hinge, then the dynamics model is established using Kane's method for the micro motion system. The optimization of the designed mechanism is investigated by using the optimization toolbox of MATLAB for multivariable nonlinear function, GA and PSO method. Moreover, four control strategies using LQR, H_2, H_∞ and mixed H_2/H_∞ method for the active vibration isolation function have been introduced. The isolation effects have been presented by using the obtained LQR, H_2, H_∞ and mixed H_2/H_∞ controllers. By using the obtained LQR, H_2, H_∞ controllers, the vibration responses of the moving platform which are decreased about 2-3 orders of magnitude are very ideal. And the mixed H_2/H_∞ controller is the best controller which can achieve a perfect performance both for the time-domain and frequency-domain with attenuating the external vibration about 15-16 orders of magnitude.

Next, an experimental system is built to implement the micro positioning and active vibration control using a 3-UPU compliant parallel manipulator with flexure hinges using two designed connecting pieces. The structures are built through mounting three PZT actuators between two flexible hinges. The micro positioning and active vibration control system is set up using National Instruments MATLAB/SIMULINK xPC target Real-Time Module. A MIMO-PID control strategy for micro positioning function has been utilized to improve the accuracy of the end-effector. The maximal tracking error can be reduced to a very ideal value. Moreover, an improved prototype with auxiliary assembly parts and new experimental setup are constructed. Eventually, active vibration control experiments are conducted for the manipulator moving with different frequency vibration, and experimental results demonstrate that the vibrations acting on the base and the moving platform are significantly reduced.

At the end of the thesis, a new 6-DOF 8-PSS/SPS compliant dual redundant platform is designed in order to improve the performance of the ultimate bearing capacity of the system. The redundant 8-PSS/SPS platform is expected to achieve either high accurate positioning or rough positioning as well as a 6-DOF active vibration isolation and excitation to the payload placed on the moving platform. The investigations will provide suggestions to improve the structure and control algorithm optimization for a compliant dual redundant parallel mechanism in order to achieve the feature of larger workspace, higher motion precision and better dynamic characteristics.

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LIST OF ABBREVIATIONS

MIMO: Multiple-Input/Multiple-Output

PZT: Piezoelectric Transducer

USM: Ultrasonic Motor

DOF: Degree Of Freedom

FPM: Flexible Parallel Mechanism

U: Universal Joint

S: Sphere Joint

P: Prismatic Joint

FEM: Finite Element Method

FPM: Flexure Parallel Mechanism

LQR: Linear Quadratic Regulator

LQG: Linear Quadratic Gaussian

LMI: Linear Matrix Inequalities

PID: Proportional and Integral and Derivative

PC: Personal Computer

NOMENCLATURE

$\mathbf{I}$:	identity matrix
$\mathbf{I}_{C_i^*}$:	inertia matrix of ith limb in global coordinate system
$\mathbf{I}'_{C_i^*}$:	inertia matrix of ith limb in the coordinate system
$\mathbf{J}$:	velocity Jacobian matrix
$\mathbf{K}_e$:	element tangent stiffness matrix
$\mathbf{K}_D$:	large-displacements stiffness matrix
$\mathbf{K}_G$:	geometric stiffness matrix
$\mathbf{K}_L$:	linear stiffness matrix
$\mathbf{K}_{\tau\ i}^{(n)}$:	tangential stiffness matrix of ith limb
$\mathbf{K}'_i$:	stiffness matrix of the whole limb in o_i-$x_iy_iz_i$ coordinate system
$\mathbf{K}'_{oi}$:	stiffness matrix of the flexure hinge connecting to the base platform in o_i-$x_iy_iz_i$
$\mathbf{K}'_{mi}$:	stiffness matrix of the flexure hinge connecting to the moving platform in o_i-$x_iy_iz_i$
$\mathbf{K}'_{ui}$:	stiffness matrix of the strut in o_i-$x_iy_iz_i$ coordinate system
$\mathbf{M}_i$:	mass matrix of ith limb
$\mathbf{M}_e$:	element mass matrix
$\mathbf{N}$:	shape function matrix
$\mathbf{H}$:	generalized density matrix
$\mathbf{T}_{ri}$:	transformation matrix of the ith limb from o_i-$x_iy_iz_i$ coordinate system to *o-xyz*
$\mathbf{F}^{m_i}$, $\mathbf{M}^{m_i}$:	force and moment exerted by the ith limb on the moving platform
$\mathbf{F}^{D}$, $\mathbf{M}^{D}$:	disturbance force and moment acting on the moving platform
$\mathbf{Q}_{m,u_r}$, $\mathbf{Q}_{C_i^*,u_r}$:	rigid body's contribution to the generalized active forces for the rth generalized speed
$\mathbf{Q}^*_{m,u_r}$, $\mathbf{Q}^*_{C_i^*,u_r}$:	rigid body's contribution to the generalized inertia forces for the rth generalized speed
$\mathbf{M}$:	mass matrix of the whole system
$\mathbf{K}$:	stiffness matrix of the whole system
$\mathbf{Q}_F$:	force matrix of the whole system
$\Re$:	rotation matrix
$\underline{\varpi}$:	disturbance vector
$\mathbf{U}$:	control vector
$\mathbf{Z}$:	performance vector
$\mathbf{Y}$:	measurement vector
$\mathbf{l}$:	vector of the three PZT actuated length variables
$\mathbf{r}_{oo'}$:	position of point o'

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$\mathbf{b}_i$:	vector $\overrightarrow{oo_i}$
$\mathbf{m}_i$:	vector $\overrightarrow{o'M_i}$
$\mathbf{d}_{oi}$, $\mathbf{d}_{1i}$, $\mathbf{d}_{2i}$, $\mathbf{d}_{mi}$:	nodal displacement vector in *o-xyz* coordinate system
$\mathbf{p}$:	external load vector acting on the moving platform
$\mathbf{p}_{oi}$, $\mathbf{p}_{1i}$, $\mathbf{p}_{2i}$, $\mathbf{p}_{mi}$:	nodal load vectors in *o-xyz* coordinate system
$\mathbf{r}_{oo'}$, $\mathbf{r}_{m_i}$, $\mathbf{r}_{C_i^*}$:	position vectors
$\mathbf{v}_{oo'}$, $\mathbf{v}_{m_i}$, $\mathbf{v}_{C_i^*}$:	velocity vectors
$\mathbf{a}_{oo'}$, $\mathbf{a}_{m_i}$, $\mathbf{a}_{C_i^*}$:	acceleration vectors
$\underline{\omega}_{C_i^*}$:	angular velocity of ith limb
$\underline{\varepsilon}_{C_i^*}$:	angular acceleration of ith limb
$\mathbf{u}$:	generalized speeds vector of the system
$\phi\left(\mathbf{d}_i\right)$:	imbalance load vector
c_i:	damping constant of ith prismatic actuator
$k_k^{j,i}$:	equivalent torsion spring stiffness constant of flexure hinge
$q_k^{j,i}$:	consecutive positive rotation angle about the kth-axis of jth flexure hinge of ith limb
$Q_k^{j,i}$:	consecutive positive rotation angle about the moved kth-axis of $\widehat{\mathbf{c}}^{ji}$
M:	mass of the moving platform
m_l:	mass of the single limb
F_i:	scalar of the actuating force of ith PZT
l_i:	length of the ith limb

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