Quasi-Monte Carlo Methods and Their Applications in High Dimensional Option Pricing

by

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A thesis submitted in partial fulfillment of the requirements for the degree of

Master of Science in Mathematics

Faculty of Science and Technology
University of Macau

2011

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Supervisor

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Abstract

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Quasi-Monte Carlo methods are revised methods of Monte Carlo methods. They both are simply and easy numerical methods to estimate the values of certain integrals. Pseudorandom numbers are used in Monte Carlo method and therefore it gives a probabilistic error bound in the valuation. However Quasi-Monte Carlo method employs deterministic sequences in the valuation. This causes the error bound to become deterministic. Furthermore, the distribution of pseudorandom numbers is not also appropriate, which may cause a decrease in accuracy in the valuation, so we need to find another sequence in which the distribution is more appropriate and controllable.

In the first chapter of this thesis, we will introduce a brief history of Monte Carlo methods as well as the basic of both the Monte Carlo methods and Quasi-Monte Carlo methods. In chapter two, we will introduce a variety of low discrepancy sequences, which will be applied in option pricing using Quasi-Monte Carlo methods. Chapter three is an introduction of different options and their pricing methods. Chapter four is a report of the numerical experiments for pricing options by using Quasi-Monte Carlo methods. In this chapter, comparison of Monte Carlo methods and Quasi-Monte Carlo methods in pricing options as well as the comparison of pricing options by Quasi-Monte Carlo methods using different low discrepancy sequences will be shown. Chapter five is a conclusion of this thesis. We found that Sobol' sequence is a good choice for Quasi-Monte Carlo method in option pricing. The relative error is rather small even in higher dimensions. With this method, theoretically the dimensions of an option being priced may be increased up to thousand.

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ACKNOWLEDGMENTS

I would like to show my gratitude to my supervisor Prof. Deng Ding, whose encouragement, guidance and support from the initial to the final level enabled me to develop an understanding of the subject. He is a great teacher. Without his downright mentorship, patience, encouragement guidance and knowledge, I would not be able to complete this thesis.

I would like to thank Prof. Xiao-Qing Jin for his fruitful lectures as well as his guidance with patient. I am also grateful to Prof. Tao Qian for his emphasis on earnest attitude towards mathematics. My thanks are also due to Prof. Che-Man Cheng, Prof. Sik-Chung Tam and Dr. Ieng-Tak Leong. I would deeply acknowledge all the staff and technicians working in the Faculty of Science and Technology for providing considerate assistance and adequate facilities to me.

Also I offer my regards and blessings to my colleagues Wai-Seng Ngan and Xin Li for their help and support in my M.Sc. journey.

Last but not least, I owe the deepest debt to my family for being the wind beneath my wings. They are, and will always be, all the matters to me.

DECLARATION

The author declares that this thesis represents his own work with Prof. Deng Ding, the author's supervisor. All the work is done under the supervision of Prof. Ding during the period 2006-2011 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.

Man-Yun Ng