

Quasi-Monte Carlo Methods and Their Applications in High  
Dimensional Option Pricing

by

Man-Yun Ng

A thesis submitted in partial fulfillment of the  
requirements for the degree of

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Approved by \_\_\_\_\_  
Supervisor

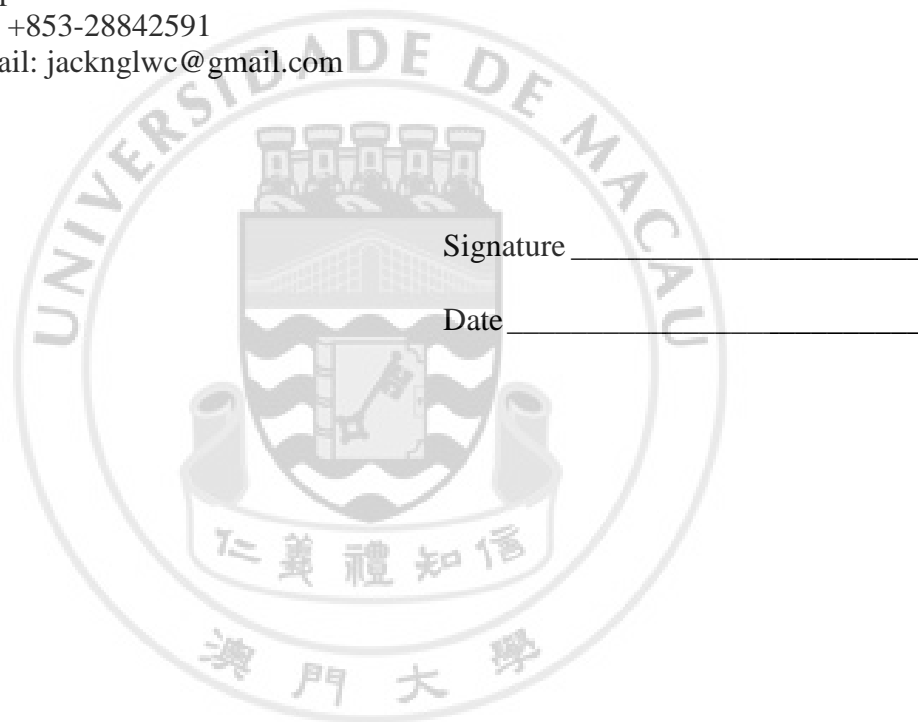
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Address: Rua de Tai Lin 412,  
edifício Lei Man,  
14 andar H,  
Taipa, Macau

Telephone: +853-66678674  
Fax: +853-28842591  
E-mail: jacknglwc@gmail.com



University of Macau

Abstract

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by Man-Yun Ng

Thesis Supervisor: Associate Professor Deng Ding  
Master of Science in Mathematics

Quasi-Monte Carlo methods are revised methods of Monte Carlo methods. They both are simply and easy numerical methods to estimate the values of certain integrals. Pseudorandom numbers are used in Monte Carlo method and therefore it gives a probabilistic error bound in the valuation. However Quasi-Monte Carlo method employs deterministic sequences in the valuation. This causes the error bound to become deterministic. Furthermore, the distribution of pseudorandom numbers is not also appropriate, which may cause a decrease in accuracy in the valuation, so we need to find another sequence in which the distribution is more appropriate and controllable.

In the first chapter of this thesis, we will introduce a brief history of Monte Carlo methods as well as the basic of both the Monte Carlo methods and Quasi-Monte Carlo methods. In chapter two, we will introduce a variety of low discrepancy sequences, which will be applied in option pricing using Quasi-Monte Carlo methods. Chapter three is an introduction of different options and their pricing methods. Chapter four is a report of the numerical experiments for pricing options by using Quasi-Monte Carlo methods. In this chapter, comparison of Monte Carlo methods and Quasi-Monte Carlo methods in pricing options as well as the comparison of pricing options by Quasi-Monte Carlo methods using different low discrepancy sequences will be shown. Chapter five is a conclusion of this thesis. We found that Sobol' sequence is a good choice for Quasi-Monte Carlo method in option pricing. The relative error is rather small even in higher dimensions. With this method, theoretically the dimensions of an option being priced may be increased up to thousand.

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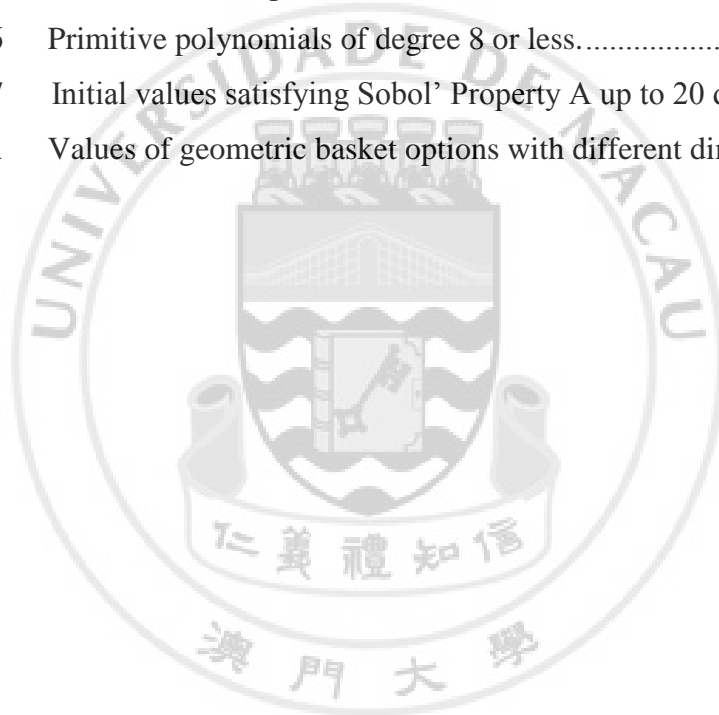
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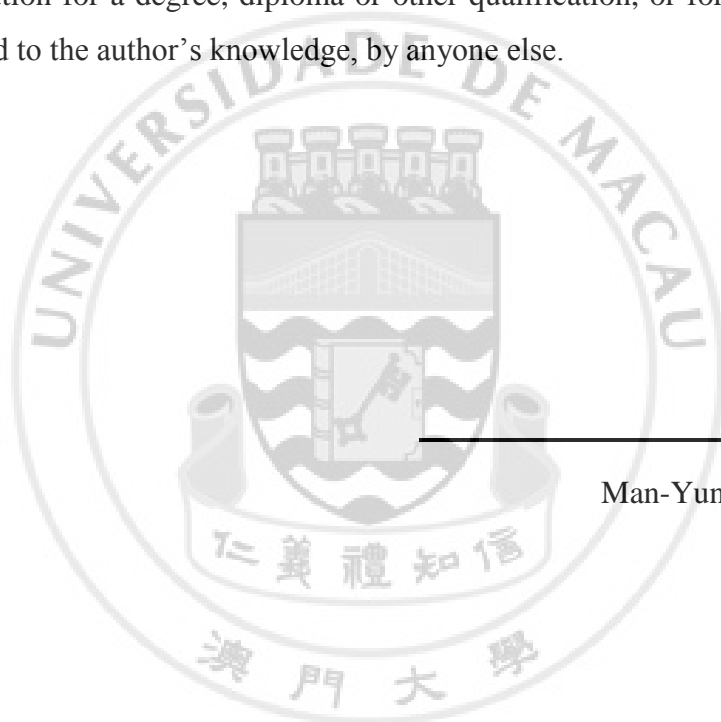
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## DECLARATION

The author declares that this thesis represents his own work with Prof. Deng Ding, the author's supervisor. All the work is done under the supervision of Prof. Ding during the period 2006-2011 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.



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Man-Yun Ng