Windowed Linear Canonical Transform and its Applications

by

XU Rui Hui

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Approved by ______________________________

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Address: Faculty of Science and Technology,

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Av. Padre Tomas Pereira, S.J. Taipa,

Macao, China.

Telephone: (853)62049018, (86)13411380661

E-Mail: axrhisme@126.com, axrhisme@gmail.com.

Signature ____________________

Date ____________________

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Thesis Supervisor:

KOU Kit Ian

Department of Mathematics

University of Macau

Introduction

The linear canonical transform (LCT) has recently received much attention in signal processing and optics [7, 9, 22]. It was first introduced in 1970s [1, 2] and is a four-parameter class of linear integral transform. The LCT is also known as the ABCD transform [10], the affine Fourier transform [11], and the generalized Fresnel transform [12]. Many operations, such as the Fourier transform (FT), the fractional Fourier transform (FrFT) [7, 22], the Fresnel transform [13], the Lorentz transform [11] and scaling operations are its special cases. With more degrees of freedom compared to the FT and the FrFT, the LCT is more flexible but with similar computation cost as the conventional FT does [14]. Due to its advantages discussed above, the LCT, as a powerful tool, has found many applications in filter design, signal synthesis, optics, radar analysis and pattern recognition, etc [7, 9]. The above mentioned applications demonstrate the great potential of LCT in signal processing. For example, filtering in the LCT domain, proposed as in [15], can achieve better performance than in the FrFT domain because of more degrees of freedom. Especially when multi-component chirp signals interfere with the desired signal, only one filter is used in the LCT domain usually, but several filters are required in the FrFT domain [16].

However, the LCT cannot reveal the local LCT-frequency contents due to its global kernel. The windowed Fourier transform (WFT) [4, 7], with a local windowed function, handles this kind of problem well. The absence of undesirable cross terms and computational simplicity result in the wide-spread use of the WFT in practice, and most other time-frequency representations can be expressed in terms of it. Nevertheless, the WFT often performs unsatisfactorily for its low resolution. The Wigner-Ville distribution maintains high localization, however it desperately suffers from spurious values in the presence of multi-components or noise [4].

Recently, some studies have attempted to attain high localization properties using fractional Fourier transform and linear canonical transform. For example, L. Stankovic et al. [24] introduces the windowed fractional Fourier transform (WFrFT) which turnes the time-frequency representations with an angle α, and then applies the WFT to it. It costs the same number of computations as realizations in the time or the frequency domain. A. Bultheel and H. Martinez-Sulbaran [8] uses a different approach, which generalizes the WFT by substituting the Fourier transform kernel with the linear canonical transform kernel in the WFT definition. It investigates some straightforward properties and two applications, the estimations of the time-of-arrival and pulsewidth of chirp signals, and the windowed fractional Fourier transform filtering. In present paper we propose the windowed linear canonical transform (WLCT), as a generalization of the latter, explore its good properties and applications.

Improving WFT by using the LCT is first proposed by H. E. Guven and O. Arikan [57]. They, through a different approach, started from basic shearing operations, generalized the group of LCT. Signals with smaller time-frequency support are represented with higher resolution, and the minimum time-bandwidth product form of the signal be achieved in several ways, either rotating the support of chirp-like signals at a suitable angle, or simply shearing it. Both operations are particular form of LCT. Here they present a generalized method for improving WFT by using LCT.

The present thesis generalize the windowed Fourier transform to the windowed linear canonical transform (WLCT), by substituting the Fourier transform kernel with the linear canonical transform kernel in the windowed Fourier transform definition. It displays the time and LCT-frequency information jointly, and is essentially a local LCT distribution, or equivalently, high resolution WFT. The WLCT offers local contents, enjoys high resolution, and eliminates cross terms.

In Chapter 1, a brief introduction of the two fundamental time-frequency operators, the LCT and WFT are given. We introduce the real WLCT in chapter 2. Some useful properties are derived, such as covariance property, orthogonality property and inversion formulas. Then the complex WLCT and high dimensional WLCT in Clifford analysis are also discussed. As applications the analogue of the Paley-Wiener theorem, uncertainty principles, Poisson summation formula, sampling formulas and series expansions. Moreover, the potential application of WLCT is also given in chapter 3. The window and parameters setting for WLCT are presented in Chapter 4. Finally, the conclusion is given in Chapter 5.

Contents

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DECLARATION

The author declares that this thesis represents his own work with Professor KOU Kit Ian, the author's supervisor. All the work is done under the supervision of Dr. KOU during the period 2009-2011 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.

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