PALEY-WIENER THEOREM AND SHANNON SAMPLING WITH THE CLIFFORD ANALYSIS SETTING

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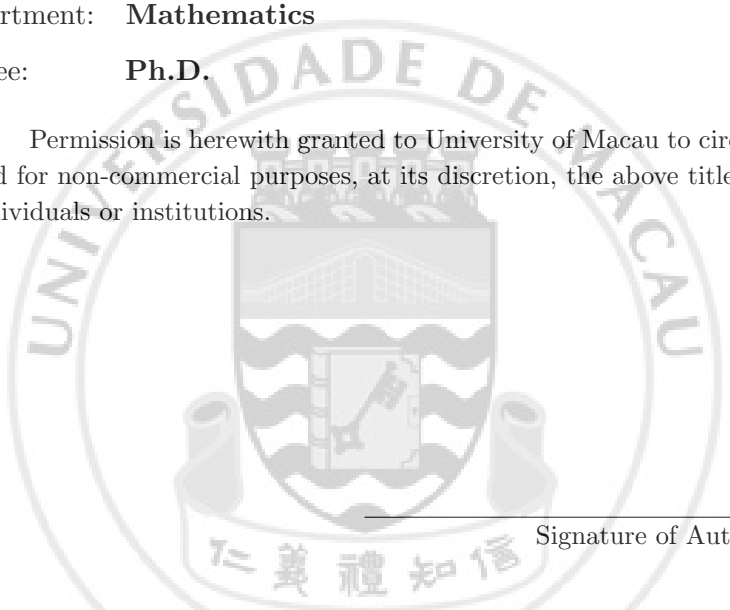

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To my parents.

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Abstract

This thesis deals with generalizations of the one-dimensional Paley-Wiener theorems and Shannon sampling theorems to Euclidean spaces with the Clifford analysis setting. Some aspects in several complex variables are also discussed. Fractional Fourier transforms in relation to Clifford-Hermite polynomials are studied. As generalizations of Fueter's Theorem, the monogenic functions of the axial type in relation to the solutions of Vekua systems are investigated.

The classical one-dimensional Paley-Wiener theorem and Shannon sampling theorems may be said to have been well understood. Motivated by theoretical and practical problems efforts have been made in order to generalize the results to higher dimensional spaces. There have been several versions for generalizations in the several complex variables setting. An effective study on these topics in $\mathbf{R}^n$ based on Clifford analysis has significant role to both mathematical concepts and innovation of techniques in analysis of several real variables.

The classical Shannon sampling theorem is based on the Paley-Wiener theorem. Therefore, generalizations of the Paley-Wiener theorem to $\mathbf{R}^n$ is of the first importance. We first successfully proved the Clifford Paley-Wiener theorem and subsequently accomplished the Clifford analogue of the Shannon sampling. In both the studies technical difficulties were encountered and overcome.

Our generalizations of Paley-Wiener and Shannon sampling theorems in $\mathbf{R}^n$ are based on a generalization of Fourier transformation. This thesis also deals with another type of generalizations of Fourier transformation. In the one-dimensional case Fourier transformation has recently been extended to a one-parameter family of unitary transformations, called fractional Fourier transformations. In collaboration with the Ghent group we obtained fractional Fourier transform results in the multi-variable case.

Much of the mentioned studies are based on producing new monogenic functions. Fueter's theorem is an important method to produce monogenic functions in the Quaternionic space. Further generalization of Fueter's theorem based on Qian's and Sommen's recent work is part of this thesis.

Results of my thesis have contributed to the following papers having appeared in the standard mathematical journals. The name order is alphabetical, except a special case. The details are as follows.

K. Kou and T. Qian, *The Paley-Wiener theorem in* $\mathbf{R}^n$ *with the Clifford analysis setting*, J. Func. Anal. **189** (2002), 227–241.

K. Kou, T. Qian and F. Sommen, *Generalizations of Fueter's theorem*, Meth. and Appl. of Anal. **9** (2002), no. 2, 273–290.

K. Kou and T. Qian, *Shannon sampling and estimation of band-limited functions in the several complex variables setting*, Acta Mathematica Scientia, Vol. 25B (2005), No. 4, 741-754.

K. Kou and T. Qian, *Shannon sampling in the Clifford analysis setting*, Zeitschrift für Analysis und ihre Anwendungen, Journal for Analysis and its Applications, Vol. 24 (2005), No. 4, 825-842.

F. Brackx, N. De Schepper, K. Kou and F. Sommen, *The Mehler formula for the generalized Clifford-Hermite polynomials*, accepted to appear in Acta Mathematica Scientia.

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