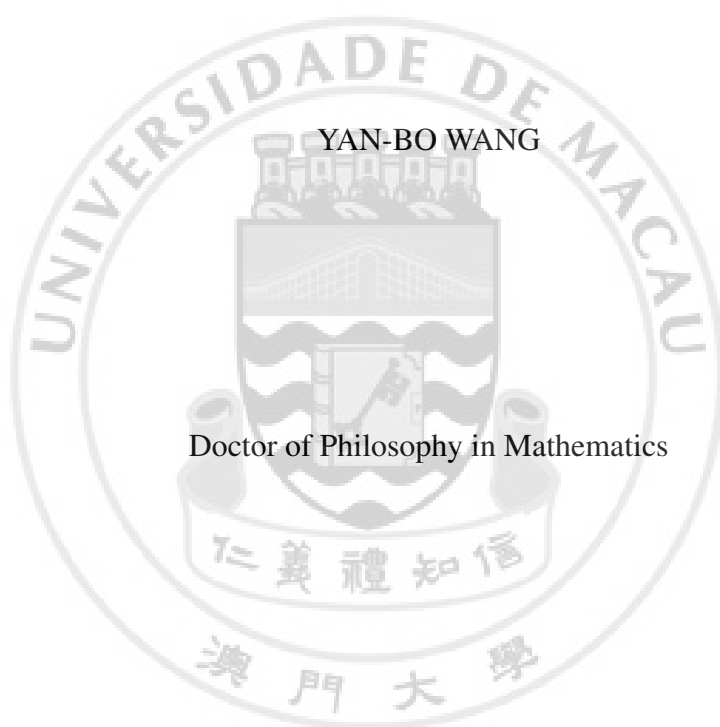


ADAPTIVE DECOMPOSITION OF SIGNALS INTO MONO-COMPONENTS

by

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Abstract

In this thesis, we study adaptive decomposition of signals into mono-components. Mono component signals are signals of non-negative analytic instantaneous frequencies which are meaningful in physics. Since the amplitude and phase are defined through analytic signals associated with real signals, real signals are automatically the real parts of analytic signals. It is natural to study complex functions in Hardy Spaces.

Mono-components are very special functions. It seems that there are two approaches to achieve decomposition of signals into its mono-components. First, one would try to obtain a decomposition only based on the signal itself. As to this field, conformal mapping may play a crucial role. In the second approach, one can seek for a large pool of mono-components and then try to decompose a signal by using the mono-components in the pool. Sufficiently many mono-components have been found including boundary values of Möbius transforms, finite and infinite Blaschke products, weighted Blaschke products, inner functions, starlike functions, etc. In our practice, we mainly work based on the second approach.

The thesis is structured as follows. Chapter 1 is an introduction. We briefly review the existing methods in signal analysis and time-frequency analysis. In Chapter 2, we give some basic concepts and description of signals, introducing analytic signals and the important concept mono-component. In Chapter 3, we review starlike functions and

study decomposition of signals into starlike functions. It is a joint work with Professor Tao Qian, Dr. Ieng Tak Leong, and Mr. Io Tong Ho. In Chapter 4, we work on adaptive decomposition of signals into T-M systems. We give the so called adaptive Fourier decomposition (**AFD**) under a maximal selection principle. It has the same spirit as greedy algorithm. We prove the convergence of **AFD** and give a convergence rate of a class of functions. From the adaptation of view, we put an insight to traditional Fourier analysis which is a special case of T-M system. It is a joint work with Professor Tao Qian. In Chapter 5, combining Nevanlinna's factorization with maximal selection principle introduced in Chapter 4, we obtain another decomposition of signals into mono-components. It is a joint work with Professor Tao Qian and Dr. Lihui Tan. In Chapter 6, we discuss remaining problems and further directions in this subject.

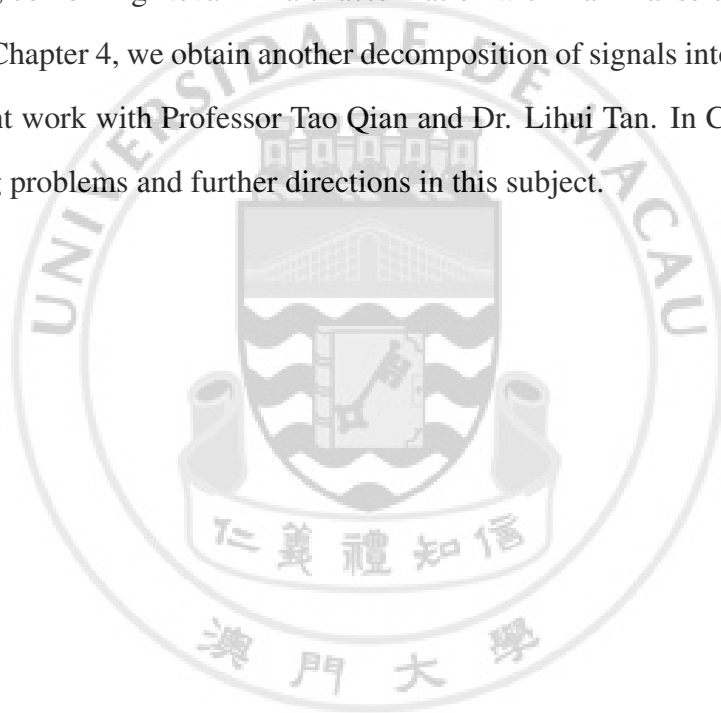


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