Time-Frequency Analysis Based on Mono-Components

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Abstract

Instantaneous frequency (IF), or analytic instantaneous frequency, is a basic and crucial concept in signal analysis. The analytic IF is defined as the phase derivative of the associated analytic signal of a signal. In general, however, a classical phase derivative of an analytic signal may not be defined, and, when it is defined, may not be non-negative. In formulating an L^2-theory for signal analysis one has to treat these obstacles, and related ones. A theoretical approach is, therefore, absolutely necessary. In this thesis, we define phase derivatives of analytic signals through non-tangential boundary limits, and consequently raise a new type of derivatives called *Hardy-Sobolev derivatives* for signals in the related Sobolev spaces. We prove that signals in the Sobolev spaces have well-defined phase derivatives that reduce to the classical ones when the latter exist. Based on the study of several types of phase derivatives and their properties, we mainly work on the following three subjects.

1. We extend the existing relations between the IF and the Fourier frequency and the related ones for smooth signals to those in the Sobolev spaces. The mentioned relations concern the mean and the higher moments of the Fourier frequency and the time represented by the instantaneous frequency and the group delay, respectively. The extended formulas in the Sobolev spaces reveal new features and new significance.

2. It has been well accepted that inner functions and outer functions in the complex analysis setting correspond to, respectively, all-pass filters and signals of minimum phase. In the existing literature of digital signal processing, however, this correspondence has never been justified for general inner and outer functions. The existing

literature are restricted to dealing with only rational functions. Based on the phase derivative theory and the recent result of positivity of phase derivatives of boundary limits of inner functions the theoretical foundation of all-pass filters and signals of minimum phase is established. Both the discrete and continuous signals cases are rigorously treated.

3. The basic objective of time-frequency analysis is to derive a two-dimensional distributional density function in the time-frequency plane for a time-varying and non-stationary signal. We study a particular type of time-frequency distribution exclusively suitable for mono-components, called *transient time-frequency distribution* (TTFD). We carry out a thorough study on the properties of TTFD. For multi-components we carry on a study on related time-frequency distributions called *composing transient time-frequency distribution* (CTTFD). A CTTFD is defined as the superposition of the TTFDs of the composing intrinsic mono-components in a suitable mono-components decomposition of the targeted multi-component signal. The suitability is interpreted as adaptivity that is a matter of art and dealt with in related work.

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