

**Finite Element Analysis of Nonlinear Stochastic Oscillators With Poisson White
Noise Excitation**

by

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**Faculty of Science and Technology
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Finite Element Method For Analyzing Nonlinear Stochastic
Oscillators With Poisson White Noise Excitation

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Abstract

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The stationary responses of some stochastic nonlinear oscillator in the presence of Poisson white noise excitations are studied in this thesis using the Finite Element Method (FEM). The FEM solution procedure is formulated and presented for the case of Poisson white noise excitation.

The probability density function (PDF) of the response of nonlinear stochastic system is governed by the generalized Fokker-Planck-Kolmogorov (FPK) when the oscillators are excited by Poisson white noise. In this thesis, FEM is further extended to obtain the approximated PDF in the presence of Poisson white noise excitations by solving the generalized FPK equation. The solution procedure for the approximate solution of the generalized FPK equation is presented. As the domain of the density function is usually infinite, equivalent statistical linearization is adopted for the estimation of a finite range of the finite element mesh. The elements with nonlinear shape functions are employed. With this method, the PDF is expressed as the product of shape functions and the node values of the density. The shape function of C^1 is constructed by the node location for the elements in local coordinates. And then 3×3 Gaussian-Legendre integration rule is used for the numerical calculation. At last a set of algebraic equations are obtained and the constraint that the integral of the probabilities over the whole domain equals 1 is considered for numerical analysis.

The solution procedure is extensively applied to different types of nonlinear oscillators with Poisson white noise excitations, including Duffing oscillators, oscillators with nonlinear damping subjected external excitations or parametric excitations or both excitations.

The numerical results have shown that the PDF results from FEM is acceptable for the cases of excitations with high impulsive arrival rate, of high excitation and of strong nonlinearity acting on displacement or velocity, or both, while the results in the tails can deviate a lot from simulation. Numerical results also have shown that FEM in the cases of parametric strong excitation acting on velocity, even the PDF can deviate a lot from simulation.



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