

Two Problems on Inverse Eigenvalue Problems and Hadamard Finite Part Integrals

by

Ka-Hang Cheang



Master of Science in Mathematics

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**Faculty of Science and Technology
University of Macau**

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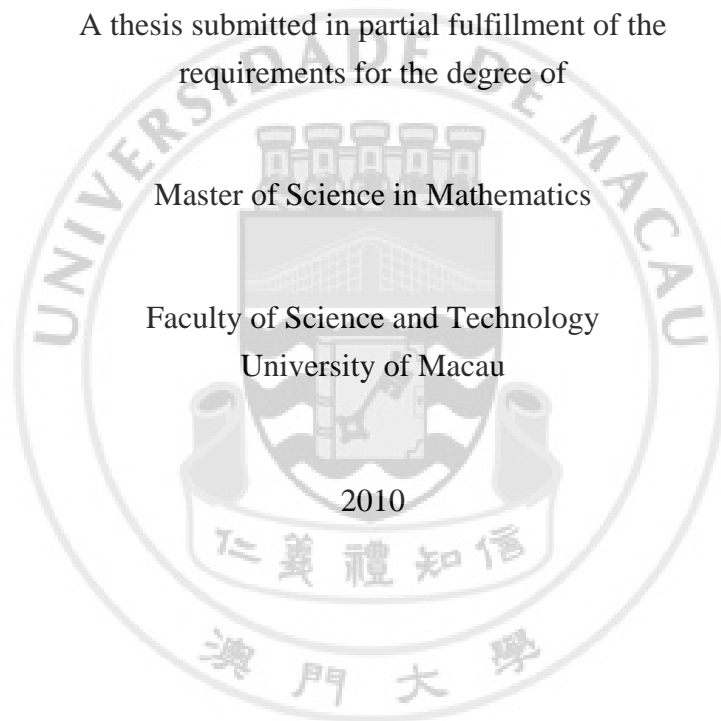
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A thesis submitted in partial fulfillment of the
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Supervisor

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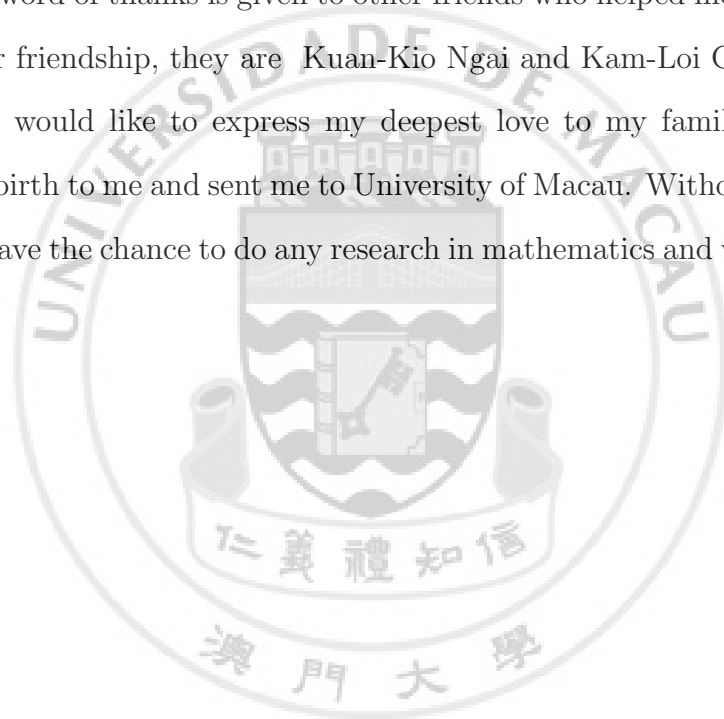
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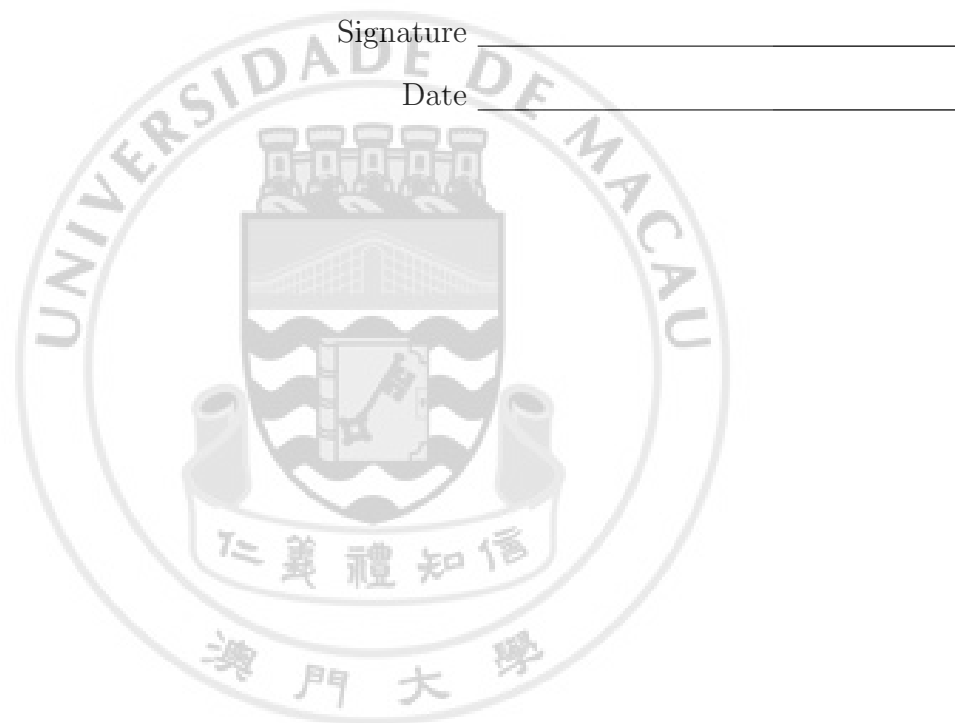
At last, I would like to express my deepest love to my family, especially my mother gave birth to me and sent me to University of Macau. Without their support, I would not have the chance to do any research in mathematics and write this thesis.



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DECLARATION

The author declares that this thesis represents his own based on the ideas suggested by Dr. Seak-Weng Vong, the author's supervisor. All the work is done under the supervision of Dr. Vong during the period 2008 - 2010 for the degree of Master of Science in Mathematics at University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.



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Abstract

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In this thesis, a brief introduction to the mathematical background of the problems under consideration is given in Chapter 1. It is well known that the conjugate gradient method is an efficient method for solving linear system. In Chapter 2, we apply the conjugate gradient method to solve the inverse eigenvalue problems, which is a nonlinear problem. We prove that the method converges locally. Numerical results show that the method converges even for some initial guesses that are not close to the true solution. In Chapter 3, the superconvergent points for the evaluation of Hadamard finite part integrals by quadrature rules are studied. People may be interested in estimating the number of superconvergent points. It has been proved that there are at most $k(-1)^k$ superconvergent points if some conditions are satisfied. Some researchers conjectured that the conditions hold in general. In Chapter 3, the conditions are proved to be true in some special cases.

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