Monte Carlo Methods in Calculating Value at Risk

by

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Supervisor

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MONTE CARLO METHODS IN CALCULATING VALUE AT RISK

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Introduction

Almost every investor who has invested or is considering investing in a risky asset will ask what is the most lose in their portfolios. VaR (Value at Risk) gives them the exact answer. Monte Carlo methods are especially useful for studying systems with a large number of coupled degrees of freedom and VaR is a typical example. In fact, the Monte Carlo methods used in the calculation of VaR allow the construction of stochastic or probabilistic financial models as opposition to the traditional static and deterministic models, so as to enhance the treatment of uncertainty in the calculation. Based on the Monte Carlo methods to compute VaR, the approach covers market conditions ranging from the general environment considered by the existing VaR methods to the financial crises which focus on stress testing. Thus, this approach evaluates the risk more accurately and advances risk awareness of investors.

This thesis reviews and summaries the calculation methods of VaR via the Monte Carlo methods. The different generations of the random numbers having normal distribution or student *t* distribution with correlations in computer are summarized and compared with each other in this thesis. The practical algorithms for calculating VaR are also presented, including the delta-gamma approximation with the important sampling. Copula technique is also presented and the practical algorithms for computing VaR via the Monte Carlo methods are also shown, including the *Gaussian* copula and the *t* copula. Finally, the numerical experiments for these different methods above are also given. From the comparison and conclusion, we can find different methods' advantages and disadvantages to estimate VaR.

In the first two chapters, the theories of VaR and the Monte Carlo methods are introduced, and their histories and applications in the finance field are also given, respectively. In Chapter 3, the theories of the different generations of the random numbers having normal distribution and *t* distribution are considered. Then the algorithms for calculating VaR via the Monte Carlo methods under those distributions are given respectively in this chapter. In Chapter 4, the theories of the generations of the random numbers using the technique of the *Gaussian* copula and the *t* copula are considered. Then the algorithms for calculating VaR via Monte Carlo methods using those copulas are given respectively in this chapter. In Chapter 5, numerical experiments for the algorithms given in Chapters 3 and 4 are shown, and the corresponding results are compared with each other by calculating these VaRs on the portfolios with different distributions or different methods, such as and the delta-gamma approximation with the important sampling.

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Glossary and Abbreviation

VaR: Value at Risk, which is the one number which can reflect the whole portfolio risk. VaR measures the potential loss in value of a risky portfolio or asset for given confidence interval over a defined period.

Gaussian **copula:** The random numbers ΔSs are generated with the *Gaussian* copula, i.e. using the Algorithm 4.1,.

t **copula:** The random numbers ΔSs are generated with the *t* copula, i.e. using the Algorithm 4.2.

ND: The random numbers ΔZs are normal distributed random variables, i.e. using the Algorithm 3.1.

TD: The random numbers ΔZs are *t* distributed random variables, i.e. using the Algorithm 3.2.

ND+IS: The random numbers ΔZs are normal distributed random variables with the important sampling, i.e. using the Algorithm 3.3.

TD+IS: The random numbers ΔZs are *t* distributed random variables with the important sampling, i.e. using the Algorithm 3.4,.

DEDICATION AND ACKNOWLEDGMENTS

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DECLARATION

The author declares that this thesis represents his own work with Professor Deng Ding, the author's supervisor. All the work is done under the supervision of Professor Ding during the period 2008-2010 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated , have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.

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