

Multigrid Algorithm Based on Cyclic Reduction for Convection Diffusion Equations

by

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Master of Science in Mathematics



**Faculty of Science and Technology
University of Macau**

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A thesis submitted in partial fulfillment of the
requirements for the degree of

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Approved by _____
Supervisor

Date _____

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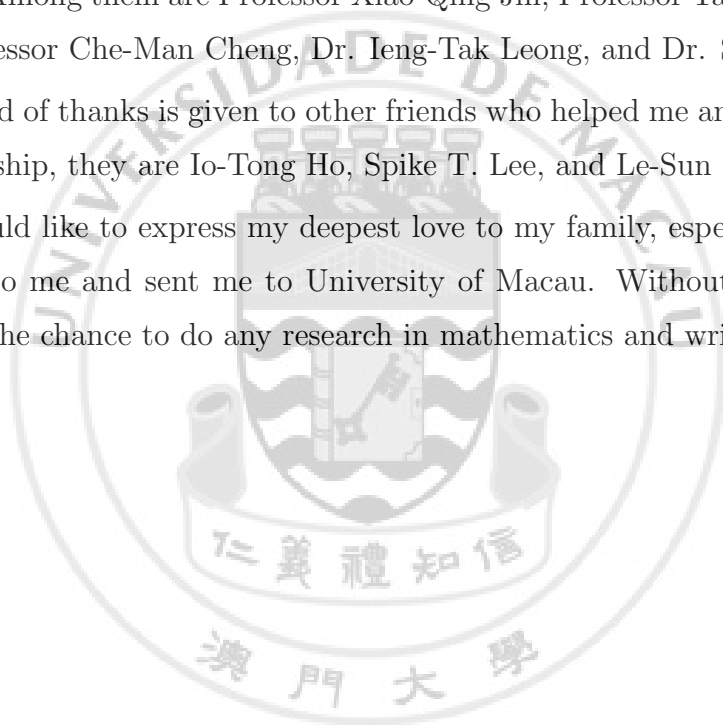
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DECLARATION

The author declares that this thesis represents his own work based on the ideas suggested by Dr. Hai-Wei Sun, the author's supervisor. All the work is done under the supervision of Dr. Sun during the period 2006 – 2010 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.

Lao Kun Leng



To

My Parents



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