Pricing Discretely Monitored Barrier Options via a Fast and Accurate FFT-based Method

By

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Supervisor

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Introduction

An efficient and precise method, which is called CONV method, for pricing Bermudan option in computational finance was proposed by Lord et al. (2008). This pricing method is based on the Fourier transformation and the convolution formula, which is dealt with numerically by using the Fast Fourier Transform (FFT). And the computational complexity is $\mathcal{O}(MN\log(N))$ with N grid points used to discrete the price of the underlying asset, and M, the number of early-exercise dates.

This thesis develops the CONV method to price discretely monitored European barrier options and Bermudan barrier options. The numerical experiments for such kinds of barrier options under different exponential Lévy models also show that the method is very fast and accurate. The total computational effort is also $\mathcal{O}(MN\log(N))$, linear in the number of early-exercise/monitoring dates M and $N\log(N)$ in the number of sample points N needed to compute the FFT.

In Chapter 1, Section 1.1 introduces the classification of options. Section 1.2 shows some different mathematical models of financial markets and gives the characteristic functions for such models. Section 1.3 summarizes the present numerical methods for pricing the barrier options.

In Chapter 2, a review of the CONV method for pricing Bermudan option is given in Sections 2.1 and 2.2, in which the key point is to discretize the convolution and the grid points. Section 2.3 gives a series of numerical experiments to verify Lord's results under the different exponential Lévy models and shows that the CONV method is very fast and accurate.

In Chapter 3, the CONV method is developed to price the discretely monitored European barrier option and the Bermudan barrier option. It's the main work of this thesis. Firstly, the recursive formulae for pricing such options can be found by backward induction. Secondly, the corresponding algorithms are shown as well. Finally, the numerical results which are given in tables and figures show that the method is efficient and superior to the other methods in some respects.

In Chapter 4, based on the results of these numerical experiments, as well as a comparison between the current method with the other two efficient methods, some conclusions about the practically numerical methods are obtained.

Contents

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