

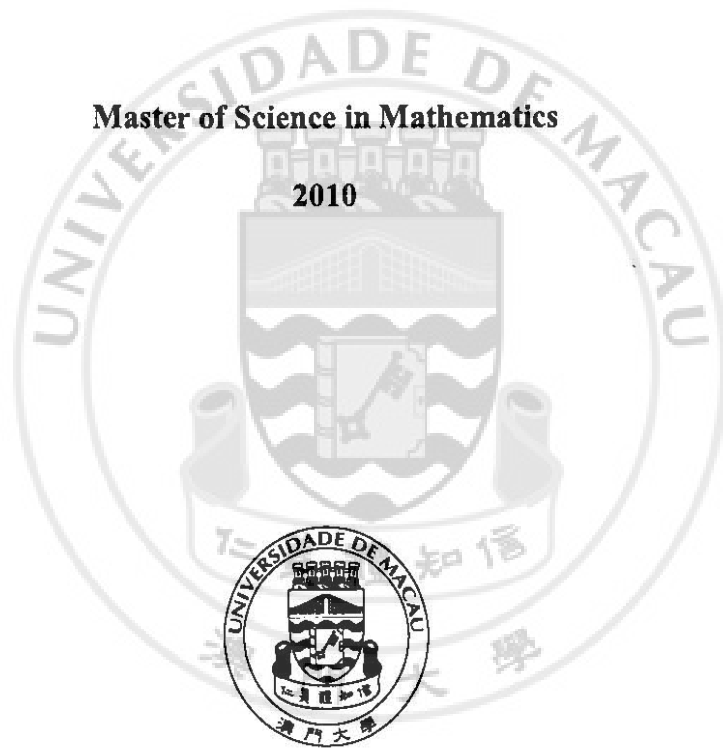
Pricing Discretely Monitored Barrier Options via a Fast and Accurate FFT-based Method

By

WENG ZUOQIU

Master of Science in Mathematics

2010



**Faculty of Science and Technology
University of Macau**

Pricing Discretely Monitored Barrier Options via a Fast and
Accurate FFT-based Method

by

WENG ZUOQIU

A thesis submitted in partial fulfillment of the
requirements for the degree of

Master of Science in Mathematics

Faculty of Science and Technology

University of Macau

2010

Approved by _____

Supervisor

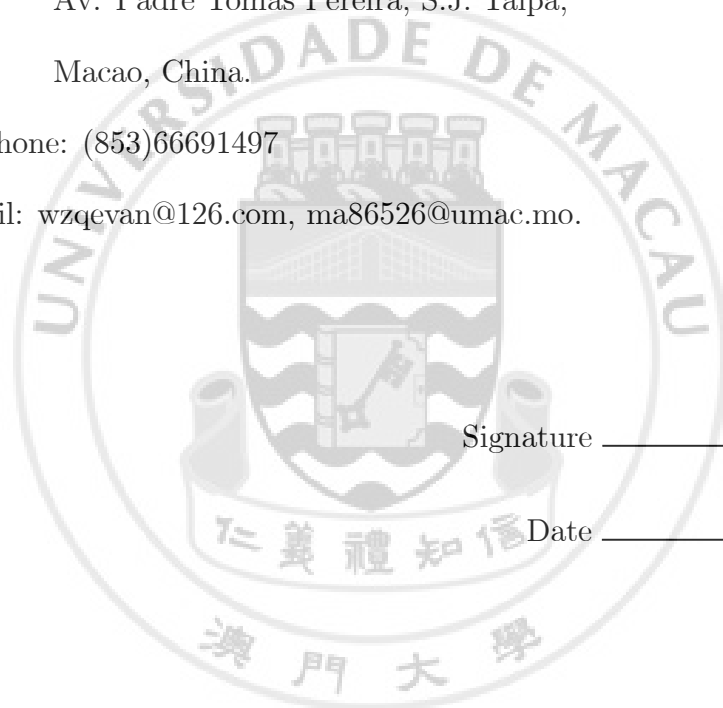
Date _____

In presenting this thesis in partial fulfillment of the requirements for a Master's degree at the University of Macau, I agree that the Library and the Faculty of Science and Technology shall make its copies freely available for inspection. However, reproduction of this thesis for any purpose or by any mean shall not be allowed without my written permission. Authorization is sought by contacting the author at

Address: Faculty of Science and Technology,
University of Macau,
Av. Padre Tomas Pereira, S.J. Taipa,
Macao, China.

Telephone: (853)66691497

E-Mail: wzqevan@126.com, ma86526@umac.mo.



Signature _____

Date _____

**Pricing Discretely Monitored Barrier Options via a Fast and
Accurate FFT-based Method**

by **WENG ZUOQIU**

Thesis Supervisor:

Professor Deng DING

Department of Mathematics

University of Macau



Introduction

An efficient and precise method, which is called CONV method, for pricing Bermudan option in computational finance was proposed by Lord et al. (2008). This pricing method is based on the Fourier transformation and the convolution formula, which is dealt with numerically by using the Fast Fourier Transform (FFT). And the computational complexity is $\mathcal{O}(MN \log(N))$ with N grid points used to discrete the price of the underlying asset, and M , the number of early-exercise dates.

This thesis develops the CONV method to price discretely monitored European barrier options and Bermudan barrier options. The numerical experiments for such kinds of barrier options under different exponential Lévy models also show that the method is very fast and accurate. The total computational effort is also $\mathcal{O}(MN \log(N))$, linear in the number of early-exercise/monitoring dates M and $N \log(N)$ in the number of sample points N needed to compute the FFT.

In Chapter 1, Section 1.1 introduces the classification of options. Section 1.2 shows some different mathematical models of financial markets and gives the characteristic functions for such models. Section 1.3 summarizes the present numerical methods for pricing the barrier options.

In Chapter 2, a review of the CONV method for pricing Bermudan option is given in Sections 2.1 and 2.2, in which the key point is to discretize the convolution and the grid points. Section 2.3 gives a series of numerical experiments to verify Lord's results under the different exponential Lévy models and shows that the CONV method is very fast and accurate.

In Chapter 3, the CONV method is developed to price the discretely monitored European barrier option and the Bermudan barrier option. It's

the main work of this thesis. Firstly, the recursive formulae for pricing such options can be found by backward induction. Secondly, the corresponding algorithms are shown as well. Finally, the numerical results which are given in tables and figures show that the method is efficient and superior to the other methods in some respects.

In Chapter 4, based on the results of these numerical experiments, as well as a comparison between the current method with the other two efficient methods, some conclusions about the practically numerical methods are obtained.



Contents

1	Introduction	1
1.1	Option Categories	1
1.1.1	Vanilla Option	2
1.1.2	Exotic Option	3
1.2	Exponential Lévy Models of Asset Prices	7
1.3	An Overview of Barrier Option Pricing	13
2	The CONV Method	18
2.1	The Recursive Formula	18
2.2	The CONV Algorithms	24
2.2.1	Discretising the Convolution	25
2.2.2	Discretising the Grid Points	29
2.3	Numerical Results	32
3	Pricing Discrete Barrier Options	35

3.1	Discrete European Barrier Option Pricing	35
3.2	Bermudan Barrier Option Pricing	38
3.3	Numerical Experiments	40
3.3.1	Numerical Results for Discrete European Barrier Option	40
3.3.2	Numerical Results for Bermudan Barrier Option	43
4	Conclusion	47
	Bibliography	49
	Appendix	53



ACKNOWLEDGEMENT

First and foremost, I wish to express my sincerest and deepest gratitude to my supervisor, Prof. Deng DING, for his inspiring guidance, support and constant encouragement throughout the whole period of my M.Sc. studies. His valuable advices and patience help me to overcome many difficulties and doubts during the preparation of this thesis.

I would also deeply acknowledge other professors who have taught me during the courses of M.Sc. studies, including Prof. Xiao-Qing JIN, Prof. Tao QIAN, Prof. Che-Man CHENG, Dr. Hai-Wei SUN, Dr. Seak-Weng VONG, Dr. Ieng-Tak LEONG, et al. I really benefited quite a lot from their guidance in academic study and suggestions on career plan.

A special word of thanks is given to other friends who helped me and encouraged me with their friendship, they are Mr. Ying-Ying ZHANG, Mr. Ning-Ying HUANG, Mr. Teng-Long WAN, Mr. Xin LIU, Mr. Jian GONG, Ms Rui-Hui XU, Mr. Zhi-Xiong LI, Ms Zhu-Lin LIU, et al.

Finally, my heartfelt thanks are dedicated to my families and my girlfriend for their constant encouragement and endless love.