

Numerical Methods for Early-exercise Option Pricing via Fourier Analysis

By

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Master of Science in Mathematics

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**Faculty of Science and Technology
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A thesis submitted in partial fulfillment of the
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Date

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Abstract

The financial market is developing explosively, although it is struck by the financial tsunami recently. Many new financial derivatives, including options, warrants and swaps are springing out. They are widely used as risk management tool by investors, stock brokers and bankers. But still, options are the most popular derivative products as hedging tools in constructing a portfolio.

Recently, Fang and Oosterlee approved a new numerical method for European option pricing which is based on the Fourier-cosine series, and called it the COS method. Then, they applied the COS method on early-exercise and discretely-monitored barrier option pricing. The main work of this thesis is developing the COS method and FFT to price Bermudan Barrier options. Some numerical experiments are done, and it works well under exponential Lévy asset price models.

In Chapter 1, Section 1.1, some basic introductions of options are given. In section 1.2, three different kind of options are introduced. In section 1.3, the mathematical background are presented.

In Chapter 2, we summarize the Fang and Oosterlee's method, and show the derivation and the algorithm for European Option, Bermudan Option and Barrier Option in Section 2.1, Section 2.2 and Section 2.3, respectively.

In Chapter 3, Section 3.1, we give the definition of Bermudan Barrier Option and derive the mathematical method to price it. In section 3.2, the corresponding algorithms are listed. This is the main work of the thesis.

In Chapter 4, Section 4.1, we do numerical experiments on pricing Bermudan Barrier Option. In section 4.2, some conclusions are summarized.

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DECLARATION

The author declares that this thesis represents his own work with Professor Deng DING, the author's supervisor. All the work is done under the supervision of Professor DING during the period 2008–2010 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.



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