Fast Exponential Time Integration Scheme and Extrapolation Method for Pricing Option with Jump Diffusions

By

Xin Liu

Master of Science in Mathematics

2010

Faculty of Science and Technology
University of Macau

Fast Exponential Time Integration Scheme and Extrapolation Method for Pricing Option with Jump Diffusions

by

Xin Liu

A thesis submitted in partial fulfillment of

the requirements for the degree of

Master of Science in Mathematics

Faculty of Science and Technology

University of Macau

2010

Approved by_____

Supervisor

_____

_____

_____

Date_____

In presenting this thesis in partial fulfillment of the requirements for a master's degree at the University of Macau, I agree that the Library and the Faculty of Science and Technology shall make its copies freely available for inspection. However, reproduction of this thesis for any purpose or by any mean shall not be allowed without my written permission. Authorization is sought by contacting the author at

Address: Faculty of Science and Technology,

University of Macau,

Av. Padre Tomas Pereira, S.J. Taipa,

Macao, China.

Telephone: (853) 83974522

Email: duetliu@gmail.com

Signature _____

Date _____

Acknowledgement

First and foremost, I wish to express my deepest gratitude to my supervisor, Dr. Hai-Wei Sun. His encouragement, support, and advice have been immensely valuable, both in personal and professional terms. I am particularly grateful to him for leading me to this interesting area of matrix exponential problems, for his emphasis on concentration on the academic research, and for the genuine concern he has shown for my development to be academic.

I am also indebted to Prof. Xiao-Qing Jin for his enthusiasm, counsel, and support throughout my M.Sc. studies. I am particularly grateful for his emphasis on international view and elegant English usage. I am specially grateful to Prof. Che-Man Cheng, Prof. Deng Ding, Dr. Seak-Weng Vong and Dr. Siu-Long Lei as well, for their enlightening suggestions, precious comments, and continual helps, from which I benefited a great deal.

My thanks are also due to other people working in the department of mathematics, for their valuable advice, warm-hearted assistance, and great support during my M.Sc. studies. Among them, I would especially like to thank Prof. Tao Qian, Prof. Sik-Chung Tam, and Dr. Ieng-Tak Leong.

I have also benefited from time spent with colleagues at the University of Macau. I am grateful to Mr. Spike. T. Lee, for his wise advice and continual helps during my study. Thanks are also given to Mr. Ying-Ying Zhang, Mr. Hong-Kui Pang, Mr. Yue-Lin Liu, Mr. Shu-Huai Xu, Mr. Ning-Ying Huang, Mr. Zuo-Qiu Weng, Mr. Jian Gong , Mr. Sio-Chong U, and Ms. Zhu-Lin Liu. Finally, I owe a great debt to my parents for their constant encouragement, financial support, and endless love.

Declaration

The author declares that this thesis represents his own work with Doctor Hai-Wei Sun , the author's supervisor. All the work is done under the supervision of Doctor Sun during the period 2008–2010 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation, or report submitted to any institution for a degree, diploma, or other qualification, or for publication by the author, and to the author's knowledge, by anyone else.

Xin Liu

Fast Exponential Time Integration Scheme and Extrapolation Method for Pricing Option with Jump Diffusions

by Xin Liu

Thesis Advisor: Doctor Hai-Wei Sun

Department of Mathematics

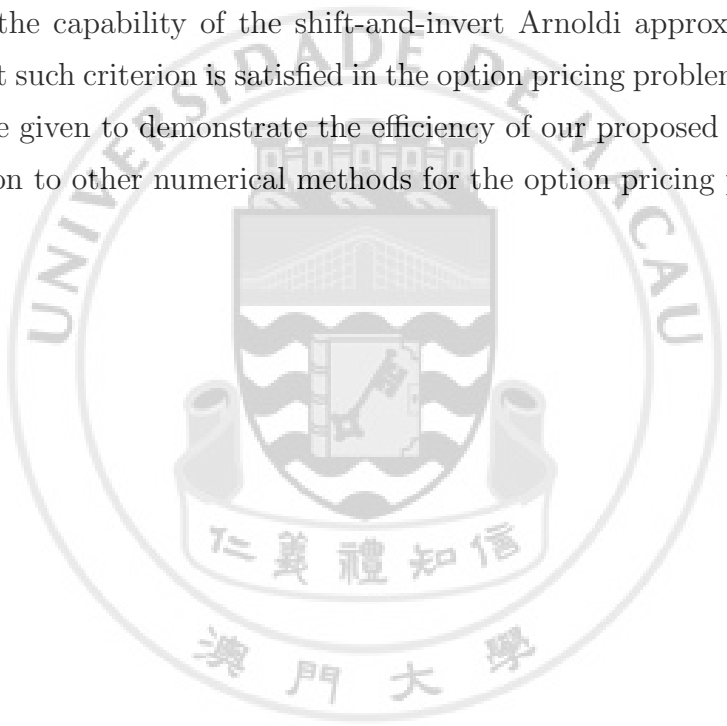
University of Macau

Introduction

In 1973, Black and Scholes [3] proposed their famous formula for pricing options under the pure-diffusion model. Later Merton [21] proposed to add lognormally distributed jumps, while Kou [18] suggested a model with double exponentially distributed jumps to improve Black and Scholes' model. In most cases, these models are treated with numerical methods. One of the numerical methods for finding option prices is related to solving a partial integro-differential equation (PIDE). For discretization of this PIDE, most existing methods employ straightforward second-order schemes for spatial direction and time-stepping schemes for time direction. Feng and Linetsky proposed to use the extrapolation approach in combination with implicit-explicit Euler (IMEX-Euler) scheme [9]. Lately, Tangman et al. [27] proposed to use an exponential time integration (ETI) scheme for handling the time direction when solving a PIDE. In Chapter 1, we mainly discuss the history of option pricing problems and numerical methods for pricing options. Among

them, we mainly focus on Feng and Linetsky's IMEX extrapolation scheme and Tangman et al.'s ETI scheme.

In [19], Lee et al. proporsed a fast approach for computing the Toeplitz matrix $[A_n]_{j,k} = a_{j-k}$ multiplied by a vector. In Chapter 2, we employs the Toeplitz matrix exponential (TME) method for pricing options. The main idea is using the shift-and-invert Arnoldi method to omit the direct computation of the matrix exponential. In this thesis, we use that method for the option pricing problem. However, the convergence analysis in [19] is not directly applicable in our case. Therefore, we propose another criterion to judge the capability of the shift-and-invert Arnoldi approximation, and prove that such criterion is satisfied in the option pricing problem. Numerical results are given to demonstrate the efficiency of our proposed scheme, with comparison to other numerical methods for the option pricing problem.



To

My Parents

Contents