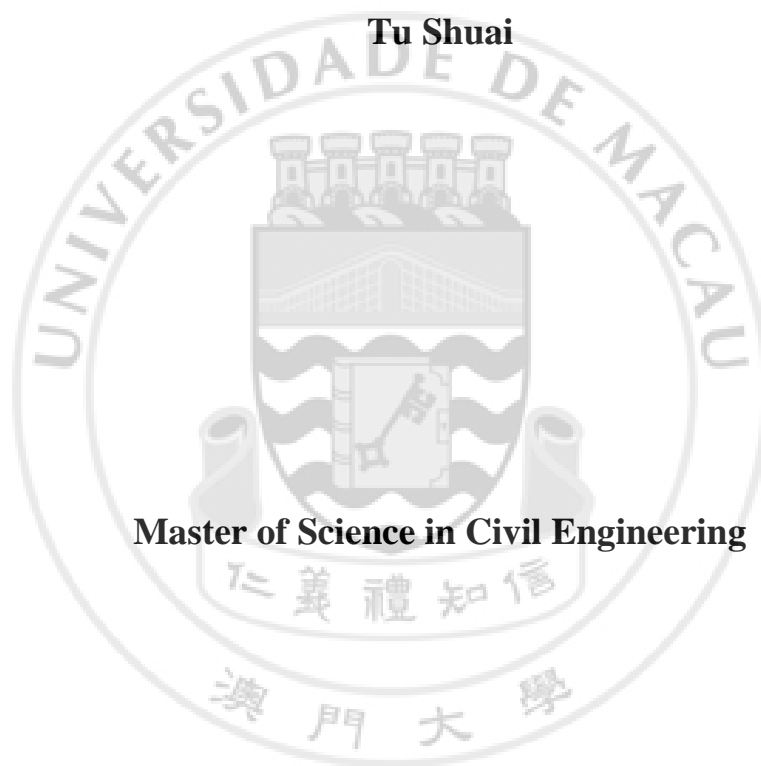


**VERTICAL DRAIN CONSOLIDATION OF
UNSATURATED SOIL BY DIFFERENTIAL
QUADRATURE METHOD**

by

Tu Shuai



Master of Science in Civil Engineering

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**Faculty of Science and Technology
University of Macau**



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A thesis submitted in partial fulfillment of the
requirement for the degree of

Master of Science in Civil Engineering

Faculty of Science and Technology

University of Macau

2013



Approved by _____ Date: _____

Supervisor

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Abstract of thesis entitled

**VERTICAL DRAIN CONSOLIDATION OF UNSATURATED SOIL BY
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by Tu Shuai

Supervisor: Dr. Zhou Wanhuan

Consolidation in a vertical drain foundation is a common problem in foundation engineering. Compared with saturated soils, unsaturated soils consist of not only water phase, but also air phase. The excess pore-water and pore-air pressures simultaneously change during the consolidation procedure and the solutions are not easy to obtain. In this thesis, the differential quadrature method (DQM) is employed to analyze consolidation of unsaturated soils in a vertical drain foundation with or without smear effect. The radial seepage of sand drain foundation is considered based on the framework of Fredlund's one-dimensional consolidation theory in unsaturated soils. With the use of Darcy's law and Fick's law, the polar governing equations of excess pore-air and pore-water pressures of consolidation in vertical drain foundation are derived. By using DQM, the two governing equations are transformed into two sets of ordinary differential equations. Then the solutions of excess pore-water and pore-air pressures can be obtained by Rong-Kutta method.

For the consolidation without smear effect, the DQ solution can be used to deal with the case of non-uniform initial pore-air and pore-water distributions. The

convergence analysis, the average degree of consolidation, the radial displacement and the vertical settlement, effects of different initial pore water/air pressures distributions and effects of different boundary conditions are presented to study the behavior of consolidation of unsaturated soils without smear effect in a vertical drain foundation.

For the consolidation with smear effect, the soil layer is divided into three parts: vertical drain, disturbed zone and undisturbed zone. In disturbed zone, as the soil around vertical drain is disturbed by the installation of vertical drain, the permeability coefficients of water and air phases decrease in some of degree compared with that in undisturbed zone. Finally, the accuracy analysis, the effect of consolidation with different permeability of water phase, the effect of consolidation with different permeability of air phase, the effect of consolidation with different length of disturbed zone are also performed to illustrate consolidation behavior of the vertical drain under smear effect are presented and discussed in this thesis.

Keywords: Axisymmetric consolidation; Unsaturated Soils; vertical drain; Differential quadrature method; Smear Effect

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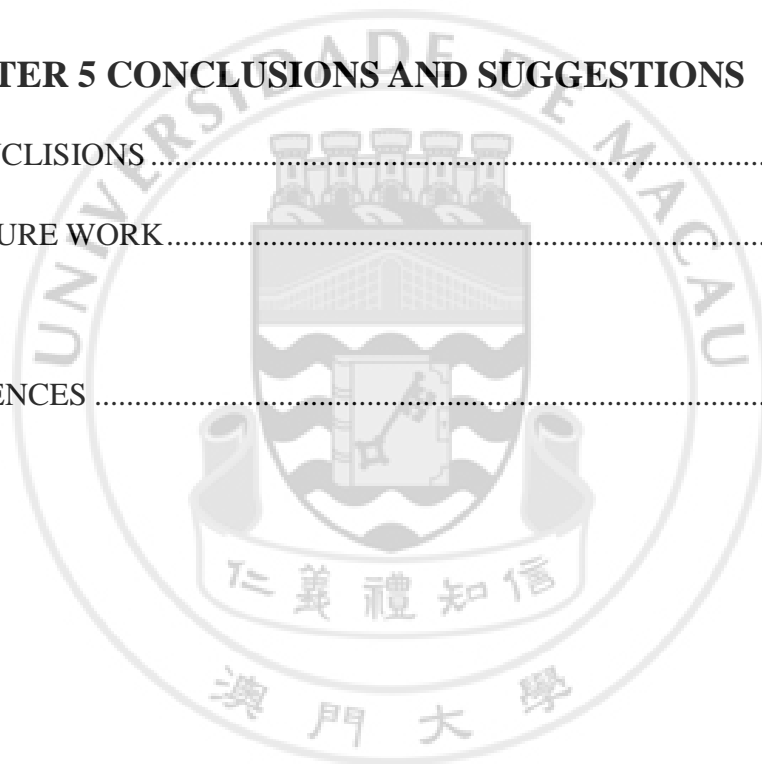
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LIST OF ABBREVIATIONS

C_v^w = The consolidation coefficient of water in consolidation without smear effect

C_v^{wi} = The consolidation coefficient of water in consolidation with smear effect

C_v^a = The consolidation coefficient of air in consolidation without smear effect

C_v^{ai} = The consolidation coefficient of air in consolidation with smear effect

$D_{ik}^{(r)}$ = The weighting coefficient

ε_v = Volumetric strain

g = The gravitational acceleration

H = The thickness of soil layer

k_a = The permeability coefficients of air in consolidation without smear effect

k_a^i = The permeability coefficients of air in consolidation with smear effect

k_a^0 = The coefficients of permeability for water at lateral boundary

k_h^a = The maximum permeability coefficients of air

k_s^a = The minimum permeability coefficients of air

k_w = The permeability coefficients of water

k_w^i = The permeability coefficients of water in consolidation with smear effect

k_w^0 = The coefficients of permeability for air at lateral boundary

k_h^w = The maximum permeability coefficients of water

k_s^w = The minimum permeability coefficients of water

m_{1k}^a = The coefficient of air volume change with respect to a change in the net normal stress

m_{1k}^w = The coefficient of water volume change with respect to a change in the net normal stress

m_2^a = The coefficient of air volume change with respect to a change in matric suction

m_2^w = The coefficient of water volume change with respect to a change in matric suction

n_0 = The initial porosity

N = The discrete point in consolidation without smear effect

N^i = The discrete point in consolidation with smear effect

ρ = The density of air phase

q = The vertical loading

r_0 = The thickness of lateral boundary

r_d = The distance from the middle of vertical drain to the lateral boundary of disturbed zone

r_e = The radius of soil layer

r_w = The radius of sand drain

r_α = The radius value of α th point

γ_w = The unit weight of water phase

R = The universal air constant

S = The distance between two sand drains

S_{r0} = The initial degree of saturation

S_v = Settlement

S_h = The radial displacement

t = Time

T = The time factor

u^a = The excess pore-air pressure in consolidation without smear effect

u^{ai} = The excess pore-air pressure in consolidation with smear effect

u^{atm} = Atmospheric pressure

u_0^a = The initial excess pore-air pressure

u^w = The excess pore-water pressure in consolidation without smear effect

u^{wi} = The excess pore-water pressure in consolidation with smear effect

u_0^w = The initial excess pore-water pressure

U^a = The average degree of consolidation with respect to air

U^w = The average degree of consolidation with respect to water

V_a = The air volume of soil layer

V_w = The water volume of soil layer

V_0 = Initial total volume of the soil