

**Analytical Solutions for Residual Stresses in Cold-Formed  
Steel Circular Hollow Sections Due to Cold Rolling**

by

**Chao CAI**

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**Faculty of Science and Technology  
University of Macau**

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