

INVESTIGATION ON EPC METHOD FOR THE PDF
SOLUTIONS OF SOME NONLINEAR STOCHASTIC
OSCILLATORS WITH POISSON EXCITATIONS

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