

THREE-DIMENSIONAL ANALYSIS OF FUNCTIONALLY  
GRADED MATERIAL PLATES – FREE VIBRATION IN  
THERMAL ENVIRONMENT AND THERMAL BUCKLING

By  
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