

# **The Girsanov's Theorems and the Novikov's Conditions**

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## **Abstract**

In this thesis, we study the Girsanov's Theorems and the Novikov's Conditions. We investigate the Girsanov's Theorems under different conditions and the various applications of them, especially in financial mathematics and stochastic differential equations. We also discuss and research different approaches for establishing the Novikov's conditions.

The thesis is divided into four chapters. In Chapter 1, basic mathematical theory is introduced. The concepts of Brownian Motions, Martingales, Stochastic Integration and Stochastic Calculus are defined. The useful theorems related to these topics are also stated in this chapter.

We start with considering the simplest case in the Chapter 2, which are the Mean Shifting Formula and Tilting Formula. Based on these ideas, we then discuss the Girsanov's Theorem for Geometrical Brownian Motions. We show that a Brownian motion with a constant drift, under a suitable change of measure, it is finally viewed as a standard Brownian motion. Furthermore, some applications of this theorem are also be illustrated.

The Girsanov's Theorem is generalized in Chapter 3 step by step. We show that the Brownian motion with a bounded drift process finally becomes a standard Brownian motion under a change of measure. We also show that if the Novikov's Condition is

satisfied, the same result still holds even the drift process is not bounded. We consider its application to general financial models in Section 3. Application to Stochastic Differential Equations is also given in the last section.

In Chapter 4, the Novikov's Condition is discussed in details. We study this theorem in different ways. We show the result by using a power series technology and a time change technology. A proof of the Novikov's Theorem under a new condition is also given in this chapter.