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Abstract

RELIABILITY ANALYSIS OF DYNAMICAL SYSTEMS USING SADDLEPOINT APPROXIMATION

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For engineering structures whose uncertainties lie in the material and geometrical properties and mechanical components as well as the loading conditions to which they are exposed during their daily operation, the assurance of performance can seldom be perfect and risk generally exists. Consequently, reliability analysis of the structures, concerned with the probability that the structure will not reach some specified state of failure, arises as a probabilistic measure of the assurance of the structural performance.

Reliability assessment has been attracting much attention for several decades in civil engineering. The problem of reliability analysis is essentially the integration of the probability density function in the failure domain. Although straightforward in principle, it is computationally prohibitive when the number of uncertain parameters is large and the failure domain is complex. Monte Carlo simulation (MCS) provides a robust alternative for this purpose. Its applicability and efficiency does not depend on the geometry of the failure domain or the number of random variables involved. However, its main drawback is that it is not efficient to estimate small failure probabilities as often required in practical applications.

The saddlepoint approximation technique is known for good approximation to the cumulative distribution function (CDF) of a random variable based on its cumulant generating function (CGF). The evaluation of the CDF value at a specified threshold requires only the process of finding one saddlepoint, where the first derivative of the CGF provides the 'threshold-saddlepoint' relationship. In this thesis, the application of the saddlepoint method in reliability analysis is discussed. In practice, a single random variable called the performance variable may be used to specify the overall response of a

structure, and the failure event is defined as the exceedance of the performance variable over a prescribed bound. According to the joint distribution of the uncertain parameters and the analysis model of the system, various sampling techniques such as MCS in the thesis can be used to generate the sample values of the performance variable. Based on the sample values, the cumulant approximation, the expectation approximation, and the proposed method as a combination of these two approaches are developed for the estimation of the CGF of the performance variable which is the key to the application of the saddlepoint method. Examples of a single-degree-of-freedom (SDOF) oscillator and a 10-story shear building subjected to random excitation are presented to examine the accuracy and efficiency of the proposed method compared with MCS.