Hilbert Transform Characterization of Boundary Values of $\mathbf{H}^2$ Functions

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Introduction

In this thesis, I mainly talk about an approach to adaptive decomposition of nonlinear and nonstable signals in signal analysis. I obtain some results on analytic signals in relation to some established theories under the frame work of T.Qian, Q.H. Chen and L.Q. Li (see [3], [4]). This thesis contains proofs of the Plemelj Theorem and the Bedrosian Theorem. I discuss boundary values of Hardy $\mathbf{H}^2$ functions and prove that for a complex-valued $\mathbf{L}^2$ function, its imaginary part is the Hilbert transform of its real part if and only if the $\mathbf{L}^2$ function is the boundary value of a $\mathbf{H}^2$ function. The method which I use is mainly the Fourier multiplier method. The counterpart theory in the unit disc is also studied.

The outline of the thesis is as follows:

Chapter 1 contains an introduction to the background knowledge of analytic signals and a survey on the Nevalina classes in the two contexts, the unit disc $\mathbf{D}$ and the upper-half plane $\mathbf{C}^+$. The two important theorems —the Plemelj Theorem and the Bedrosian Theorem are proved in this chapter. We include a concise introduction to Nevanlina class in relation to the Hardy spaces in §1.3.

Chapter 2 and chapter 3 are, respectively, devoted to the theories in the unit disc and in the upper-half complex plane.

Chapter 2 deals with the theory in the unit disc. We first introduce the known results which can be found in [3] and then give a new result, Theorem 2.2, in relation to $\mathbf{H}^2$ spaces.

In chapter 3, we use the Fourier multiplier method to obtain a new result, Theorem 3.1, in upper-half complex plane. The main reference is [5].