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Abstract

**BACKWARD STOCHASTIC DIFFERENTIAL  
EQUATIONS AND PRICING**

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This thesis is concerned with backward stochastic differential equations (BSDEs) and their application to European option and American option pricing problems. Unlike the traditional pricing, the option pricing does not depend on the costs of production in the past, but depends on the price of the stock on which the option is written in the future. Moreover, in general, the price of stock is a stochastic process. Hence, it is natural and direct, and so it is also useful and powerful, to study the option pricing problem by using BSDE. Since the general BSDE were first introduced by Pang and Pardoux in 1990, the BSDE appear numerous problems in the option pricing theory. This thesis considered reflected BSDE, and then applied these formulas to discuss the pricing and hedging problems of European options and American options.

In Chapter One, the basic definitions and explanations of financial derivatives, options and option pricing were introduced and discussed, and a brief classifications of options were introduced.

In Chapter Two, basic concepts and important results of stochastic calculus were reviewed and summarized. A brief introduction of stochastic differential equations was given. The BSDEs, including linear BSDEs and linear reflected BSDEs, were considered. The closed formulas of solutions of a linear BSDE and a linear BSDE were shown.

In Chapter Three, as a comparison, the Black-Scholes model for option pricing was introduced and discussed. The uniqueness of hedging strategies for European options was proved. The result for explicit solution of a linear BSDE was applied to

find the hedging strategy and to price a European option in an Itô's financial model.

In Chapter Four, the hedging and pricing problems of American options were considered and discussed. The call option pricing problem and the put option pricing problem were solved respectively. In special, the result on solution of reflected BSDE is applied for the solving of American put option pricing problem.