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Abstract

Martingale Method in Option Pricing Theory

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This thesis is concerned with the martingale method in option pricing theory. The martingale method is based on the stochastic analysis and modern martingale theory, and was developed in the 90s of last century. Because the stochastic calculus, which includes the modern martingale theory, plays an important role in option pricing theory, the martingale method provides us a natural and powerful approach to analyze the value process of options (or other financial derivatives), and to derive their pricing formula. Moreover, this method also provides us a useful approach to study the pricing problem in a more general financial model, not only in the Itô's model. This thesis summarized and extended this martingale method in option pricing theory, and by this method discussed and studied the mathematical meanings and representations of some basic concepts in option pricing theory.

This thesis is organized as follows. In Chapter One, basic definitions and explanations of financial derivatives, options and option pricing are introduced and discussed, and some prerequisites in probability theory are reviewed.

In Chapter Two and Chapter Three, A brief classifications of options are introduced. By consideration of a simple binomial model, the basic concepts and the basic ideal of option pricing theory are introduced and illustrated. As a comparison, by the classical methods, the Black-Schoes equations and the Black-Schoes formula are introduced and derived.

In Chapter Four and Chapter Five, some basic concepts and important results of stochastic calculus, including martingale theory are reviewed and summarized. In specially, the martingale representation theory and the Girsanov's theorem are introduced and discussed. The financial meaning of equivalent martingale measure was deeply discussed. A result about pricing a European option by the martingale method is shown, and then, a formula for option pricing is obtained. The martingale method was extended to a more general financial model, whose stock price process is a semmartingale. After a brief introduction of stochastic calculus based on semimartingales, the option hedging and pricing problems are discussed under some general hypotheses.