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Abstract

Convergence of Fourier Series on the Sphere

in the Clifford Analysis Setting

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In this thesis, we establish a new approach to study pointwise convergence of Fourier-Laplace series on unit spheres in Euclidean spaces $\mathbf{R}^n$.

We introduce a Clifford frame to $\mathbf{R}^n$, which is, in a great extent, similar to the complex structure of the one dimensional complex plane. In this sense, Fourier-Laplace series on unit spheres can be reduced to Fouries series on the unit circle.

We then restrict ourselves to the Hamilton Quaternionic space, which is a special case of general $\mathbf{R}^n$ and $\mathbf{R}_1^n$ spaces. Based on [9], we seek for a new approach on the unit sphere in the quaternionic space. The basic technique is to use Fueter's result to establish a corresponding relationship between complex Laurent series and quaternionic Laurent series, as well as between their Dirichlet kernels representing partial sums.

By virtue of this correspondence, we obtain the expression and estimate of the Dirichlet kernel for the quaternionic space. Finally, we show that using our approach in the quaternionic space some new pointwise convergence theorems can be obtained by reducing to the similar argument in the unit circle case.