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Abstract

Distance to the Convex Hull of an Equivalence Class by Special Orthogonal Equivalence

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In this thesis, we study an approximation problem in the $n \times n$ real matrix space $M_n(\boldsymbol{R})$. Two matrices $A, B \in M_n(\boldsymbol{R})$ are said to be special orthogonal equivalent if there exist $n \times n$ special orthogonal matrices U and V such that $A = UBV$. A norm $\| \ \|$ on $M_n(\boldsymbol{R})$ is said to be special orthogonally invariant if

$$\|UAV\| = \|A\| \quad \text{for any } n \times n \text{ special orthogonal matrices } U, V.$$

Let $B/_{\sim}$ be the equivalence class of B by special orthogonal equivalence and let $\operatorname{conv} B/_{\sim}$ denote its convex hull, i.e., the smallest convex set containing $B/_{\sim}$.

Given $A, B \in M_n(\boldsymbol{R})$ and a special orthogonally invariant norm $\| \ \|$, we determine the quantities

$$\max\left\{\|A - X\| : X \in \operatorname{conv} B/_{\sim}\right\}$$

and

$$\min\left\{\|A - X\| : X \in \operatorname{conv} B/_{\sim}\right\}.$$

It turns out that we can find matrices B_M and B_m in $\operatorname{conv} B/_{\sim}$ such that

$$\|A - B_m\| \le \|A - X\| \le \|A - B_M\| \qquad \text{for all } X \in \operatorname{conv} B/_{\sim},$$

and the pair of matrices work for all special orthogonally invariant norms.

The problem is ultimately amount to solving an approximation problem in R^n, which is of independent interests.