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Abstract

Normal Systems of Modal Logic

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This thesis is an introductory text in modal logic, the logic of necessity and possibility; being the main purpose the study of the normal systems, that are presented from an axiomatic standpoint.

It is divided into three chapters. Chapter 1 introduces the meaning of modal and describes a concise historical idea about modal logic and its development since Aristotle till nowadays.

Chapter 2- Logical Preliminaries- consists of eight sections, meant to furnish a sufficient background and understanding of modal logic. Sections 2.1 and 2.2 contain the syntactic and semantic concepts, respectively. As an application, the validity of some sentences is proved in section 2.3. Section 2.4 gives the relationship between modal logic and propositional logic, showing that the modal logic includes the propositional one. Some examples of invalidity are shown in section 2.5. The three last sections are dedicated to systems of modal logic, including the first definitions and results, maximality and, as an illustration, the examination of **S5**, one of the best-known systems of modal logic.

Chapter 3- Normal systems of Modal Logic- including five sections, contains an extensive study of the referred systems from an axiomatic standpoint, as said before. Section 3.1 contains the first definitions and properties of normal systems and are given alternative bases for normal systems; some, necessity- based, other, possibility- based. Different characterizations are proved in section 3.2 and several schemas are proved to be theorems of any normal system (theorem 3.8). In section 3.3, modally generalization, replacement and duality are presented as important tools; as shown, for instance, in the proof of theorem 3.12. The normal extensions of **K** (the smallest normal system of modal logic) obtained by adding as theorems the schemas **D**, **T**, **B**, **4**, and **5** are rather extensively studied in sections 3.4 and 3.5, as important examples of normal systems of modal logic.