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Abstract

The Existence of Solutions to 1-Dimensional Strictly Hyperbolic Conservation Laws

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In this thesis, we study 1-dimensional strictly hyperbolic conservation laws. One typical example of hyperbolic conservation laws is the isentropic gas dynamics system in Lagrangian coordinates. This system is quasilinear. For nonlinear hyperbolic equations, solution may become discontinuous even for smooth initial data. Moreover, since the characteristics of the system depend on the solution, they may become equal as time elapses and the system becomes non-strictly hyperbolic. In this case, the system is nondiagonizable and little has been known for nondiagonizable system. The isentropic gas dynamics system becomes non-strictly hyperbolic when the specific volume becomes infinity. "The problem of the appearance of the vacuum in these equations has been a central issue in this field for some time because without the vacuum, the equations that model compressible fluid flow admit no bounded invariant regions. The issue of vacuum states has been addressed by many of the modern leaders in this field, including Diperna, Smoller and Liu." (J. Temple) After L.W. Lin's result in 1987, people believe that in general, vacuum does not occur at finite time. However, at the present stage, it seems that our knowledge about the solution is still too weak to prove this conjecture. We

want to know more about the structure of the solution. In chapter 2 of this thesis, we study the qualitative behavior of waves and shows that interaction between two centered rarefaction waves is an extreme case among all kinds of interactions when the boundary values and support of the waves involved are fixed. As an application, we use this result to prove that vacuum does not occur for rarefaction flow even though the initial flow is not piecewise Lipschitz continuous.

Although many results have been obtained for the isentropic gas dynamics system in Lagrangian coordinates, little has been known for the non-isentropic gas dynamics system. No framework has turned out to be appropriate for the general study of the non-isentropic gas dynamics system and only partial results have been built. In 1989, C.Z. Li and J.H. Wang constructed a global classical solution by pasting three simple waves together. For the polytropic gas, in the same year, Y.C. Zhao used a maximal principle to give a sufficient condition that the system admits global classical solution, but this condition contradicts one of his assumptions in the proof. In 1997, L.W. Lin, H.X. Liu and T. Yang gave a sufficient condition for the existence of global continuous solution for the polytropic gas, but the solution is still non-classical in the sense that the derivative is only piecewise continuous. In 1999, F.G. Liu modified Y.C. Zhao's maximal principle to give a condition for the system to admit global classical solution. However, the author gives a counter-example for his modification in chapter 3 of this thesis. We find a system with initial data satisfying his condition but the conclusion fails. Since Liu's proof based on this modification, so the proof is not vaild. Although C.J. Zhu has come up with the existence of global classical solution, however, he assumes that the state equation $p(v, s)$ is an exponential function of specific volume. Hence, up to the present, whether global classsical solution exists for polytropic gas still remains as an open question for further studies.