

MIS (C)
013
FAN

**Control of Double Link Inverted Pendulum with
Nonlinear Friction**

by

Fang Lei

July 1999



**Electrical and Electronics Engineering
Faculty of Science and Technology
University of Macau**

TABLE OF CONTENTS

LIST OF FIGURES.....	vi
LIST OF TABLES	viii
CHAPTER 1: Introduction.....	1
1.1 Prelude.....	1
1.2 Brief History of Inverted Pendulum Systems	2
1.3 Problem Formulation and Thesis Goal.....	6
1.4 Thesis Organization.....	7
CHAPTER 2: Model Development and System Description	8
2.1 Prelude.....	8
2.2 Equations of Motion for Multi-Link Inverted Pendulum.....	9
Static Friction Models	13
New Multi-Link Dynamic Equations.....	15
2.3 Double Link Inverted Pendulum System Description.....	20
2.4 Physical Parameters Identification	22
Identification of Links' Parameters	23
Identification of Parameters of Cart System	24
CHAPTER 3: Control Systems Design by State Space Mehods	28
3.1 Prelude.....	28
3.2 Swing Up Control Design	28
3.3 Regulator Design.....	31
Selection of Weighting Matrices.....	33
3.4 Servo Controller Design.....	34
3.5 Observer Design.....	35
3.6 Implication of Observer's Use on Real-time Control	39
3.7 Time-Multiplied Quadratic Performance Regulators.....	40
CHAPTER 4: Friction Models and Friction Compensation	43
4.1 Prelude.....	43

4.2 Discontinuous Friction Models	44
4.3 An Alternate Friction Model	45
4.4 Friction Compensation by Dither	46
4.5 Model-based Compensation for Friction	51
CHAPTER 5: Case Study - Double Link Control	54
5.1 Prelude	54
5.2 Linear Quadratic Regulator Design	55
5.3 Digital Control Implementation	61
Estimation of Unmeasurable State Variables	62
System Responses and Performance	64
Modified Integral Control of Cart Position	66
5.4 Sample Rate Selection	72
5.5 Compensating Friction of Cart System	73
CHAPTER 6: Conclusions and Future Work	80
6.1 Prelude	80
6.2 Conclusions	80
6.3 Future Work	82
REFERENCE	84
APPENDIX A: System Matrices and Parameters	89
APPENDIX B: Double Inverted Pendulum Apparatus	91
APPENDIX C: Source Codes	93