Uncertainty principles for quaternionic linear canonical transform and applications

by

Jian-Yu Ou

A thesis submitted in partial fulfillment of the requirements for the degree of

Master of Science in Mathematics

Faculty of Science and Technology

University of Macau

2013

Approved by

Supervisor

Date
In presenting this thesis in partial fulfillment of the requirements for a Master’s degree at the University of Macau, I agree that the Library and the Faculty of Science and Technology shall make its copies freely available for inspection. However, reproduction of this thesis for any purpose or by any mean shall not be allowed without my written permission. Authorization is sought by contacting the author at

Address: Faculty of Science and Technology,
University of Macau,
Av. Padre Tomas Pereira, S.J. Taipa,
Macao, China.
Telephone: (853) 62992275.
Email: mb15493@umac.mo.

Signature _______________________
Date ________________________
Uncertainty principles for quaternionic linear canonical transform and applications

by Jian-Yu Ou

Thesis Supervisor:

Kit-Ian Kou

Department of Mathematics
University of Macau

Abstract

In terms of this influential but rarely discussed theory, this thesis seeks to explore and generalize the linear canonical transform (LCT) to quaternion-valued signals. We call it the quaternionic linear canonical transform (QLCT).

In a communication theory setting, an uncertainty principle states that a signal cannot be arbitrarily confined in both the spatial and frequency domains. Many efforts have been devoted to extend the uncertainty principle to various types of functions and linear canonical transforms. We establish an uncertainty principle for the QLCT by using the properties of the LCT and describe a lower bound on the product of the effective widths of quaternion-valued signals in the spatial and frequency domains by applying the uncertainty principle established in the first part, pointing out that only a Gaussian quaternionic signal can minimize the uncertainty.
One of the basic problems encountered in signal representations using the conventional LCT is the ineffectiveness of the LCT kernel to represent and compute location information. One method to overcome such a problem is the windowed Linear canonical transform (WLCT). Following this method we define windowed quaternionic linear canonical transform (WLCT). The QWLCT has the similar properties with QLCT. Finally we established uncertainty principle of QWLCT.
# Contents

1. Introduction .......................................................... 1

2. Quaternion Algebra .................................................. 4

3. Linear canonical transform of 2D quaternionic signals ........... 8
   3.1 LCTs of 2D quaternionic signals ................................. 8
      3.1.1 Definition .................................................. 9
      3.1.2 Properties .................................................. 10
   3.2 QLCTs of 2D quaternionic signals ............................... 17
      3.2.1 Definition .................................................. 17
      3.2.2 Properties .................................................. 21
   3.3 Uncertainty principles for QLCTs ................................. 26

4. Quaternion window linear canonical transform of 2D quaternionic signals 37
4.1 QWLCT of 2D quaternionic signals

4.1.1 Definition of the QWLCT

4.1.2 Examples of the QWLCT

4.1.3 Properties of QWLCT

4.2 Heisenberg’s uncertainty principle for the QWLCT

5 Conclusion

Bibliography
ACKNOWLEDGEMENTS

First and foremost, I wish to express my sincerest and deepest gratitude to my supervisor, Dr. Kit-Ian KOU, for her support and inspiring guidance throughout my master’s study. I thank her for showing me the beauty of mathematic analysis, teaching me the art of writing research papers, for her support on many research-related matters and her encouraging advice on my research.

I would also deeply acknowledge other professors during my master’s study, including Prof. Tao QIAN, Prof. Xiao-Qing JIN, Prof. Deng DING, Prof. Hai-Wei SUN, Prof. Seak-Weng VONG, Dr. Siu-Long LEI, Prof. Huai-Dong CAO, Prof. Sik-Chung TAM etc. I had really benefited quite a lot from their classes and out-class-communication with them.

I want to give my warmest thanks to my dear friends, officemates and colleagues, for all the helps they gived to me: Mr. Wei-Xiong MAI, Ms. Zhu-Lin LIU, Ms. Yan MO, Ms. Wen-Yan JIA, Ms. Hai-Lin Zhao, Mr. Pan-Qiu XIA, Mr. Le-Ping LIANG, Ms. Shu-Lin LV, Ms. Yuan GAO, Mr. Zhi-Bo WANG, Mr. Jian-Hao LV, Mr. Ze-Zhen CAO, etc.

Last but not least, my heartfelt thanks are dedicated to my parents.
DECLARATION

The author declares that this thesis represents her own work with Dr. Kit-Ian Kou, the author’s supervisor. All the work is done under the supervision of Dr. Kit-Ian Kou during the period 2011-2013 for the degree of Master of Science in Mathematics at the University of Macau. The results in this thesis, unless otherwise stated or indicated, have not been previously included in any thesis, dissertation or report submitted to any institution for a degree, diploma or other qualification, or for publication by the author, and to the author’s knowledge, by anyone else.

Jian-Yu Ou