

Introduction

Many systems of equations presented in the science and engineering area, which are the differential equations mostly, can be solved numerically by finite difference method (FDM). As well-known, the FDM is the most efficient method to obtain the numerical approximation of the differential equations and its main idea is that we are able to obtain the numerical approximation on the discrete numerical grids instead of the original continual domain by using finite difference scheme. For partial differential equations (PDEs), we are able to obtain the the numerical approximation by replacing the partial derivatives by finite differences on the discrete numerical grids.

The content of the FDM includes how to discretize the original continual domain to grids; how to generate the difference equations and solve them. If we take the regular discrete numerical grids, the finite difference scheme is the only part we have to consider. The trend toward highly accurate numerical schemes of PDEs has recently led to a renewed interest in compact difference schemes. Concurrently, Hirsh[1] and Kreiss[2] have proposed Hermitian compact techniques using less nodes (three instead of five) at each grid point to solve PDEs. Later on, as pointed out by Adam[3], the truncated errors are usually four to six times smaller than the same order noncompact schemes. Since then, much work has been done in developing compact schemes for various applications, such as: an implicit compact fourth-order algorithm; a fourth-order compact difference scheme for nonuniform grids[4]; fourth-order and sixth-order compact difference schemes for the staggered grid; an early form of the sixth-order combined compact difference scheme; compact finite difference schemes with a range of spatial scales[5]; and an upwind fifth-order compact scheme. These schemes are characterized by (a) five-point sixth-

order, (b) much lower accuracy at nodes adjacent to boundaries and (c) no requirement on PDEs to be satisfied at boundaries.

Several recent work emphasizes on the improvement of boundary accuracy. For hyperbolic system, Carpenter et al.[6,7] introduced a simultaneous approximation term (SAT) method that solves a linear combination of the boundary conditions and the hyperbolic equations near the boundary. This method provides fourth-order accuracy at both interior and boundary. Under the assumption that the derivative operator admits a summation-by-parts formula then the SAT method is stable in the classical sense and is also time-stable. For 2D vorticity-stream function formulation, E and Liu[8,9] proposed a finite difference scheme with fourth-order accuracy at both interior and boundary. Those schemes are not good enough. We keep looking for a scheme with two more things (1) working for any differential equation and (2) having high-order accuracy at both interior and boundary.

A three-point sixth-order and eighth-order combined compact difference (CCD) scheme is such a scheme with the following features: (a) three-point sixth-order and eighth-order, (b) comparable accuracy at nodes adjacent to boundaries and (c) requirement on PDEs to be satisfied at boundaries. Fourier analysis of errors is used to prove the CCD scheme as having better resolution characteristics than any current (uncompact and compact) scheme. Furthermore we give two examples to demonstrate the benefit of using CCD scheme.