

Analysis on GMRES Convergence and Some Results on Spectral Properties of Preconditioned Matrices

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Introduction

In this thesis, we analyze the convergence rate of the GMRES method by the Chebyshev polynomial. Also, we study some spectral properties of preconditioned matrices. The thesis is divided into three chapters. Some basic theory of the GMRES method and some preconditioners are introduced in Chapter 1.

In Chapter 2, we analyze the upper bound for the GMRES iterations by the Chebyshev polynomial. We show that the number of the GMRES iterations required for convergence approaches to a constant as the center of the ellipse c tends to infinity. We also show that the number of iterations increases with at most $\mathcal{O}(\log n)$ where n is the matrix size if the matrix satisfies some conditions. Numerical results to show the convergence rate of the GMRES method are also given.

In Chapter 3, we study the diagonalization and stability property of the optimal preconditioned matrices $c_F^{-1}(A_n)A_n$ with $A_n \in \mathbb{C}^{n \times n}$, and the stability property of $c_F^{-1}(H_n)A_n$ where $H_n = (A_n + A_n^*)/2$. We also give relations

between extreme eigenvalues of optimal and superoptimal preconditioned matrices. Some examples are also given to illustrate this result.