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Abstract

Martingale Representation Theorems and Their Applications

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The Martingale Representation Theory is an important tool in the stochastic analysis. This theory consists of martingale representation theorems, which can be found in many textbooks or literatures. But they usually appear in various form and are always proved without details. It makes a confusion for beginner or researchers. The purpose of this thesis is to summarize and compare these theorems, to give complete and detailed proofs of these theorems. Some applications of these theorems are also provided in this thesis.

Dudley's representation theorem tells us that any Brownian random variable can be represented as the stochastic integral of a suitable function. Brownian Martingale representation theorem tells us that a Brownian continuous martingale can represent as the stochastic integral of a suitable function. Lévy's theorem interpretive of the stochastic integral as time change of Brownian motion. Using the general martingale theory, the Lévy's theorem can be extended to general martingales. Continuous martingale representation theorem tells us for any continuous and square integrable

martingale; we can find a Brownian motion to represent this continuous martingale as a stochastic integral.

Finally, these theorems will be applied to get Brownian motion characterization, and to find the solution of backward stochastic differential equations. In financial mathematics, these theorems also play an important role in the arbitrage pricing of derivative securities.