Some Properties of T. Chan's Preconditioner and Applications in Numerical Differential Equations

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Abstract

In this thesis, we study some properties of T. Chan's preconditioner and apply T. Chan's preconditioner in solving linear systems deduced from numerical differential equations.

In Chapter 1, we introduce a relative new method, which is called Boundary Value Method (BVM). When this method is applied to discretize differential equations, one can get a non-symmetric linear system $\mathbf{Hy} = \mathbf{b}$. To solve such a linear system, the generalized minimal residual (GMRES) method combined with preconditioner is usually used. Some basic results of these methods and preconditioners are given in this chapter.

T. Chan's preconditioner's properties is discussed in Chapter 2. A matrix is said to be stable if the real part of all the eigenvalues are negative. For some class of matrices, we show that T. Chan's preconditioner is stable. By using this result, we prove the invertibility of some preconditioners proposed in numerical ordinary differential equations (ODEs) recently. Some numerical experiments which deduced from ODEs are given to show the effectiveness of our methods.

In the last Chapter, the numerical solution of delay differential equations (DDEs) is considered. By applying BVM, one can get some nonsymmetric,

large and sparse linear systems. These systems are solved by using the GMRES method. A mixed-type block-circulant preconditioner with circulant-blocks (BCCB preconditioner) is proposed to speed up the convergence rate of the GMRES method. We prove that if an $A_{\nu,k-\nu}$-stable BVM is used and the system matrices of delay differential equations are normal, then the BCCB preconditioner is invertible. The advantage of using such type of preconditioners is that one can use 2-dimensional Fast Fourier transform to reduce the operation cost. Numerical results are given to show the superiority of the BCCB preconditioner.