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Abstract

Approximation theory of multilayer feedforward artificial neural network

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ANN is a system loosely modeled on the human brain. Its high efficiency and accuracy in numerable ways over other traditional methods makes it be applied widely in multiple areas. However, a mathematical theoretical base sufficiently powerful to support the ANN system has been expected over a long period of time. Actually, many mathematicians work hardy to fill up this lacuna and many of them obtained considerate results. Secondly, discussions about the contribution of this mathematician are helpful to the development of applications of ANN.

Our study is carried out around the "Approximation Capabilities of Multilayer Feedforward Networks"[1], an excellent contribution in the mathematical studies for multilayer feedforward networks. Our main purpose is to prove in very detail the 1^o- 2^otheorems presented in the paper mentioned above (these theorems stated that: 1. The ANNs with bounded and nonconstant functions are universal aproximators with respect to arbitrary finite input environment measure μ . 2. The ANNs with continuous bounded and non-constant transfer functions can learn uniformly over compact input sets.) and we also justify its applicability in general multilayer feedforward networks.

Our studies contains three chapters. In the first chapter, we present the mathematical model of neuron and an index system for the parameters and variables of the ANN that we use through out our thesis. We also explain the function of the mathematical model of neuron in detail. Our work in this chapter is to facilitate our studies in store for it.

In the chapter two, we prove the uniqueness theorem of Fourier transform for measures on $\mathbb{R}^n$, which is indispensable for our studies in the chapter 3. For this purpose, we consider the LCA group and measures defined on it, the topological structure of the

dual group and Fourier transform for measures and essentially the characteristic of dual group of $\mathbb{R}^n$.

In the chapter 3, we finally prove the theorems which constitute the target of our studies and justify its applicability in general multilayer feedforward networks. To deal with it, the canonical network will be defined as a simplify version of multilayer feedforward network.