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Abstract

Fast Transform Based Operators for Toeplitz Systems and Their Applications in Image Restoration

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In this thesis, we study the fast transform based operators and their applications in some large-scale image restoration problems. The fast transform based operator maps every arbitrary matrix $\mathbf{A}_n$ to the optimal transform based preconditioner. This preconditioner is defined to be the minimizer of $\|\mathbf{V}_n - \mathbf{A}_n\|_F$ over a set of matrices $\mathbf{V}_n$ that can be diagonalized by a fast discrete transform matrix. We discuss the algebraic and geometric properties of the operators. Then we consider the spectral properties of the operators. Numerical tests are also given.

We then study the solutions of block Toeplitz systems $\mathbf{A}_{mn}\mathbf{x} = \mathbf{b}$ by the preconditioned conjugate gradient (PCG) method. Here $\mathbf{A}_{mn} = \mathbf{A}_m \otimes \mathbf{A}_n$ and $\mathbf{A}_i, i = m, n$ are Toeplitz matrices. By applying the PCG method, Jin [23] introduced a fast algorithm with the optimal circulant preconditioners for solving these systems. This fast algorithm allows a tensor problem to be reduced to a one-dimensional problem. It was proved that if the $mn \times mn$ system is well-conditioned, then the PCG method converges superlinearly and only $\mathcal{O}(mn \log mn)$ operations are required in solving the preconditioned system.

However, only well-conditioned systems were considered in [23]. Therefore we will apply this fast algorithm with the $\{\omega\}$-circulant preconditioner proposed in [31] to manage the ill-conditioned systems. Numerical results are included to illustrate the effectiveness of this fast algorithm for solving the preconditioned systems by using the PCG method.

Finally, the image restoration problems are introduced. Actually, many image restoration problems are ill-posed, so it is difficult to find the required solution. Regularization is commonly used to cope with these problems. We will use the Tikhonov technique to deal with such ill-posed problems. Numerical experiments with different optimal transform based preconditioners are reported to illustrate the effectiveness of these preconditioners for solving the image restoration problems.